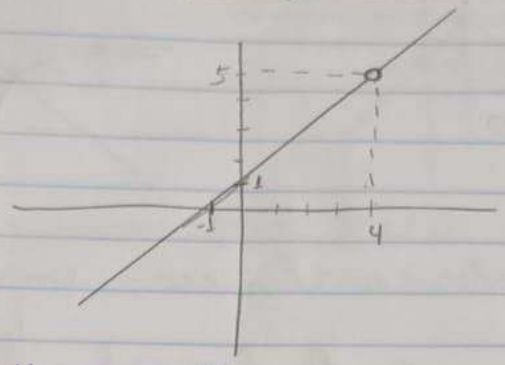
Seção 2.6 2, 4, 8, 9, 14, 16, 20, 24, 26, 33, 38 e 39

2) $f(x) = \frac{x^2-3x-4}{x-4}$ $D = \mathbb{R} - \{4\}$

$x^2 - 3x - 4 \mid x - 4$
$-x^2 + 4x \quad\quad x + 1$
$\quad x - 4$
$\quad -x + 4$

$\lim_{x \to 4} = \frac{x^2-3x-4}{x-4} \Rightarrow \lim = \frac{(x-4)(x+1)}{(x-4)}$

$\Rightarrow \lim_{x \to 4} x + 1 = 5$



$y = x + 1$
$-x + y = 1$
$-y = 1$
$x = -1$

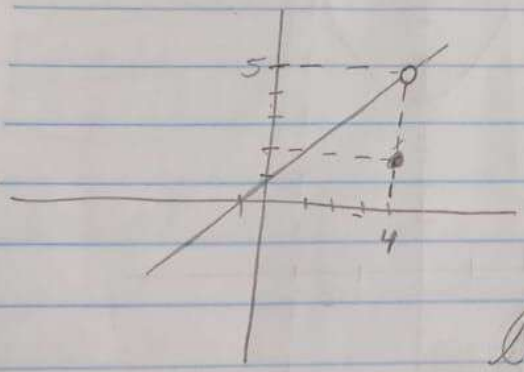
4. $g(x) = \begin{cases} \frac{x^2-3x-4}{x-4} & \text{se } x \neq 4 \\ 2 & \text{se } x = 4 \end{cases}$

$g(4) = 2$ ✓

$\lim_{x \to 4} \frac{x^2-3x-4}{x-4} \Rightarrow \lim_{x \to 4} \frac{(x-4)(x+1)}{(x-4)}$

$\lim_{x \to 4} x + 1 = 5$ ✓

$\lim_{x \to 4} g(x) \neq g(4)$



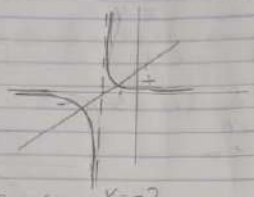
Logo g é descontínua em $x = 4$

8) $g(x) = \begin{cases} \frac{1}{x+2} & \text{se } x \neq -2 \\ 0 & \text{se } x = -2 \end{cases}$ $\quad g(-2) = 0$

$\lim_{x \to -2} \frac{1}{x+2}$ = não existe

$\lim_{x \to -2^-} \frac{1}{x+2} \overset{\frac{1}{0^-}}{=} -\infty$

$\lim_{x \to -2^+} \frac{1}{x+2} \overset{\frac{1}{0^+}}{=} +\infty$



Logo $g(x)$ é descontínua em $x = -2$

9) $f(x) = \frac{x^4 - 16}{x^2 - 4}$ $\quad D = \mathbb{R} - \{-2, 2\}$

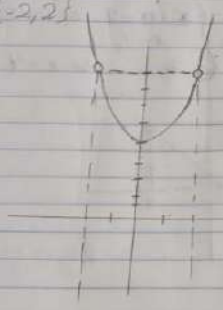
$\lim_{x \to 2} \frac{(x^2-4)\cdot(x^2+4)}{(x^2-4)} \Rightarrow$

$\Rightarrow \lim_{x \to 2} x^2 + 4 = 8$ ✓

$\lim_{x \to -2} x^2 + 4 = 8$ ✓

$y = x^2 + 4$

$y - x^2 = 4 \quad y = 4$



Logo f é descontínua em $x = 2$ e $x = -2$.

14) $f(x) \begin{cases} x - 1 & \text{se } x < 1 \\ 1 & \text{se } x = 1 \\ 1 - x & \text{se } x > 1 \end{cases}$ $\quad f(1) = 1$

$\lim_{x \to 1^+} 1 - x = 0$

$\lim_{x \to 1^-} x - 1 = 0$

$\Big\} \lim_{x \to 1} f(x) = 0$

Logo, f é descontínua em $x = 1$

16) $H(x) = \begin{cases} 1 + x & \text{se } x \leq -2 \quad (-3, 1) \\ 2 - x & \text{se } -2 < x \leq 2 \quad (2, -1) \\ 2x - 1 & \text{se } 2 < x \quad (2, 3) \end{cases}$

$y = 1 + x$ $\quad y = 2x - 1$ $\quad y = 2x - 1$

$y = 1$ $\quad 2x - y = 1$ $\quad -2x + y = -1$

$x = -1$ $\quad x = \frac{1}{2}$

$(0, 1)$ $\quad -2$

$(-1, 0)$ $\quad \boxed{x = \frac{1}{2}}$

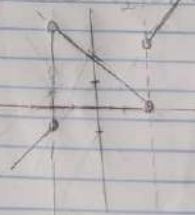
$y = 2 - x$ $\quad (\frac{1}{2}, 0) \quad (0, -\frac{1}{2})$

$y = 2$

$x = 2$

$(0, 2)$

$(2, 0)$



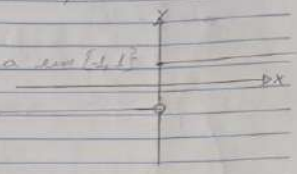
Logo $H(x)$ é descontínua

20. $g(x) \begin{cases} |x| & \text{se } x \neq 0 \\ 1 & \text{se } x = 0 \end{cases}$ $\quad x=0$ $\quad y=1$

$|x| = \begin{cases} y=-1 \\ y=1 \end{cases}$

$\lim_{x \to 0} |x| = 0$ não existe

$\lim_{x \to 0} x = 0$



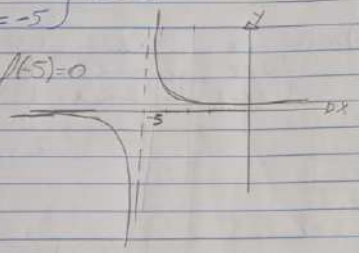
Logo $g(x)$ é descontínua em $x=0$

$g(0) = 1$ ✓

24) $f(5) = \begin{cases} \frac{1}{5+5} & \text{se } 5 \neq -5 \\ 0 & \text{se } 5 = -5 \end{cases}$ $\quad a = -5$

$\lim_{5 \to -5^+} \frac{1}{5+5} \overset{\frac{1}{0^+}}{=} +\infty$ $\quad f(-5) = 0$

$\lim_{5 \to -5^-} \frac{1}{5+5} \overset{\frac{1}{0^-}}{=} -\infty$



Essencial.

26) $f(x) = \frac{x^2-x-12}{x^2+2x-3}$; $a = -3$

$D = \mathbb{R} - \{-3, 1\}$

$\lim_{x \to -3} \frac{x^2-x-12}{x^2+2x-3} =$

$\lim_{x \to -3} \frac{(x+3)(x-4)}{(x+3)(x-1)} = \frac{-3-4}{-3-1} = \frac{-7}{-4} = \frac{7}{4}$

como $\lim_{x \to -3} f(x) = \frac{7}{4}$, logo a descontinuidade em $x=-3$ é removível

$f(x) = \begin{cases} \frac{x^2-x-12}{x^2+2x-3}, & \text{se } x \neq -3 \\ \frac{7}{4}, & \text{se } x = -3 \end{cases}$

$\Delta = b^2 - 4ac = 49$

$x = \frac{7 \pm 12}{2}$

$x = 16$

$\frac{-2 \pm 4}{2} = \frac{4}{2} = 2$ $\quad -3$

33) $f(x) = x^2(x+3)^2 \Rightarrow x \cdot (x+9) \Rightarrow x^4 + 9x^2$

como $f(x)$ é uma função polinomial f é contínua em todos os números reais.

38) $g(x) = \frac{x-2}{x^2+2x-8}$ $\quad D = \mathbb{R} - \{-4, 2\}$

$2^2 - 4 \cdot 1 \cdot -8$ $\quad +2, \frac{4}{-2}$

$4+32$ $\quad 1^\circ, -4 = -8$

$\Delta = 36$

$\frac{-2 \pm 6}{2}$ $\quad \frac{4}{-2} = -4$

Logo $g(x)$ uma função g é contínua em todos os números de seu domínio, logo g é contínua em todos os números reais exceto em $x=-4$ e $x=2$.

39) $f(x) = \begin{cases} 2x-1 & \text{se } x < 2 \\ 4-x^2 & \text{se } 2 \leq x \end{cases}$

$y = 2x-1$ (0,-1)
$y = -1$
$y = 2x-1$
$-2x = -1$ (1,0)
$x = \frac{1}{2}$

$y = 4-x^2$
$x = 0$
$x^2 = 4$
$x = \pm 2$

$2x-1 = 5$
$x = 3$

$D = \mathbb{R} - \{2, -2\}$

$f(x)$ é contínua em todos os reais menos no $x=2$

Seção 2.7 .01, 08, 09, 15, 18, 23
do 35 ao 38 fazer 1
do 45 ao 48 ; fazer 1
do 49 ao 56 . fazer 1 a cada 4
fazer 61 ou 62.

1) $f(x) = \sqrt{x}$; $g(x) = 9-x^2$
$h = f \circ g$ $h = \sqrt{9-x^2}$ $D = \mathbb{R} - [-3, 3]$

Logo h será contínua em todos os números do intervalo $(-3, 3)$.

8) $f(x) = \frac{1}{x^2}$; $g(x) = x+3$

$h = f \circ g \Rightarrow h = \frac{1}{(x+3)^2}$ $h = \frac{1}{x^2+9}$ $D = \mathbb{R} - (-3)$

Logo h será contínua em todos os reais menos no $x = -3$

9) $f(x) = \sqrt{x}$; $g(x) = \frac{1}{x-2}$ $h = \sqrt{\frac{1}{x-2}}$ $h = \frac{1}{\sqrt{x-2}}$

$D = \mathbb{R} - (2)$

Contínua em todos os números reais de $x = (2, +\infty)$

15) $f(x) = \frac{2}{x+5}$ $D = \mathbb{R} - (-5)$ $]-\infty, -5[\cup]-5, +\infty[$

a) $(3,7)$ não tem -5 no intervalo é contínua
b) $[-6,4]$ tem -5 no intervalo descontínua
c) $[-5,+\infty)$ inclui o -5 descontínua
c) $(-\infty, 0)$ tem -5 no intervalo é contínua
d) $(-5, +\infty)$ começa em -5 não é contínua
e) $[-10, -5]$ termina no -5 contínua

18) $f(x) = \frac{x+3}{x^2-4}$ $D = \mathbb{R} - [2, -2]$ $]-\infty, -2[\cup]2, +\infty[$

a) $(0,4]$ descontínua
b) $(-2,2)$ contínua
c) $(-\infty, -2]$ descontínua
d) $(2, +\infty)$ contínua
e) $[-4,4]$ descontínua
f) $(-2,2]$ descontínua

23) $f(x) = \sqrt{4-x^2}$

$4 - x^2 \geq 0$

$D = \mathbb{R} - [-2, 2]$

a) $(-2, 2)$ contínua

b) $[-2, 2]$ contínua

c) $[-2, 2)$ contínua

d) $(-2, 2]$ contínua

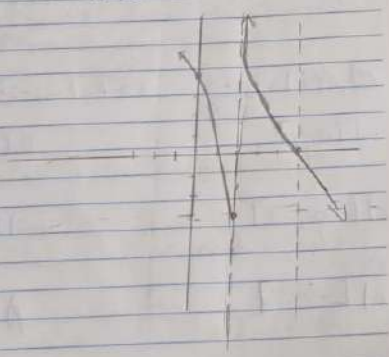
e) $(-\infty, -2]$ descontínua

f) $(2, +\infty)$ descontínua

35) f é contínua em $(-\infty, 2]$ e $(2, +\infty)$:

$\lim_{x \to 0} f(x) = 4$; $\lim_{x \to 2^-} f(x) = -3$; $\lim_{x \to 2^+} f(x) = +\infty$

$\lim_{x \to 5} f(x) = 0$



45) $f(x) = \begin{cases} 3x+7 & \text{se } x \leq 4 \\ kx-1 & \text{se } x > 4 \end{cases}$

$\lim_{x \to 4^+} kx - 1 = 4x - 1 = 4 \cdot 5 - 1 = 19$

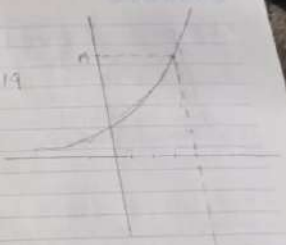
$\lim_{x \to 4^-} 3x + 7 = 19$

$4k - 1 = 19$

$4k = 20$

$k = \frac{20}{4}$

$k = 5$



49) $f(x) = 2 + x - x^2$; $[a, b] = [0, 3]$; $k = 1$

$a = -1$ $\quad \Delta = 1 - 4 \cdot -1 \cdot 1 \quad x = \frac{-1 \pm \sqrt{5}}{2}$

$b = 1$ $\quad \Delta = 1 + 4$

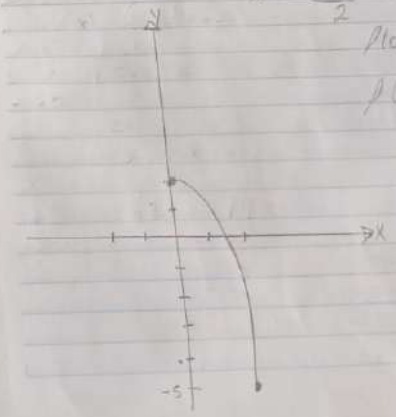
$c = 1$ $\quad \Delta = 5$

$x' = \frac{-1 + \sqrt{5}}{2} = -\frac{1}{2} + \frac{\sqrt{5}}{2}$

$x'' = \frac{-1 - \sqrt{5}}{2} = -\frac{1}{2} - \frac{\sqrt{5}}{2}$

$f(0) = 1 + 0 - 0 = 1$

$f(1) = 1 + 1 - 1 = 1$



Exercícios 3.1 Do 13 ao 20 fazer os a cada 04
Do 21 ao 24 fazer os a cada 02
fazer 25, 30, 31, 35, 36
Do 43 ao 46 fazer 3
do 47 ao 50 fazer os
do 51 ao 56 fazer os a cada 03.

(13) $y = x^2 - 4x - 5 ; (-2, 7)$

$$m = \lim_{x \to 0} \frac{(x+\Delta x)^2 - 4(x+\Delta x) - 5 - (x^2 - 4x - 5)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - 4x - 4\Delta x - 5 - x^2 + 4x + 5}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2 - 4\Delta x}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta x (2x + \Delta x - 4)}{\Delta x}$$

$$\lim_{\Delta x \to 0} 2x + \Delta x - 4$$

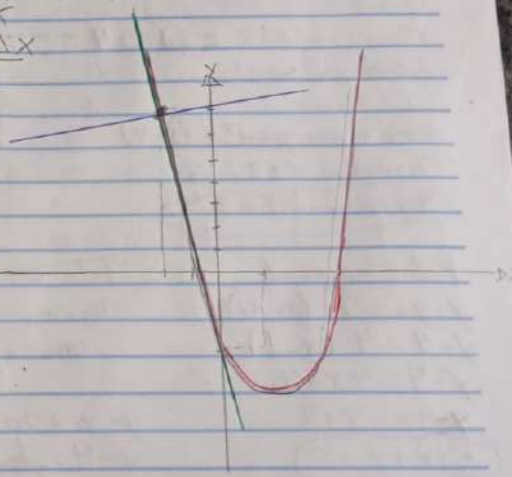
$$\lim_{\Delta x \to 0} 2x - 4 = 2.(-2) - 4 = -8$$

$m(x) = 2x - 4$

no ponto $(-2, 7)$

$m_x = 2.(-2) - 4$

$m_x = -8$



Reta Tangente

$y - y_0 = m_x (x - x_0)$

$y - 7 = -8 (x + 2)$

$y - 7 = -8x - 16$

$y + 8x - 7 + 16 =$

$\boxed{8x + y + 9 = 0}$ Rt

reta normal

$m_x \cdot m_n = -1$

$-8 \cdot m_n = -1$

$m_n = \frac{-1}{-8}$

$m_n = \frac{1}{8}$

$y - y_0 = m_n (x - x_0)$

$y - 7 = \frac{1}{8}[x - (-2)]$

$8y - 56 = x + 2$

$-x + 8y - 56 - 2$

$\boxed{-x + 8y - 58 = 0}$ Rn

16) $y = x^2 + 2x + 1$; (1, 4)

$m = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 + 2(x+\Delta x) + 1 - (x^2+2x+1)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 + 2x + 2\Delta x + 1 - x^2 - 2x - 1}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\Delta x^2 + 2x\Delta x + 2\Delta x}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\Delta x(\Delta x + 2x + 2)}{\Delta x}$

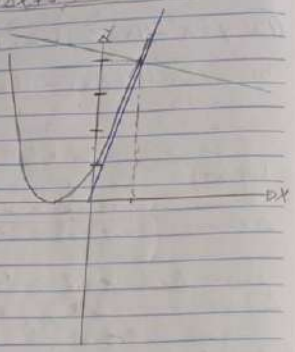
$\lim_{\Delta x \to 0} \Delta x + 2x + 2$

$\lim_{\Delta x \to 0} 2x + 2 = 2.1 + 2 = 4$

m_x = do ponto (1, 4)

$m_x = 2x + 2$

$m_x = 2.1 + 2 = 4$



Reta tangente

$y - y_0 = m(x - x_0)$

$y - 4 = 4(x - 1)$

$y - 4 = 4x - 4$

$y - 4x - 4 + 4$

$-4x + y = 0$

Reta normal

$m_t \cdot m_n = -1$

$m_n = -\frac{1}{4}$

$y - y_0 = m(x - x_0)$

$y - 4 = -\frac{1}{4}(x - 1)$

$4y - 16 = -x + 1$

$x + 4y - 17 = 0$

$\Delta = 4^2 - 4.1.1$

$\Delta = 4 - 4$

$\Delta = 0$

$x = \frac{-2 \pm 0}{2} \quad \frac{-2}{2} = -1$

18) $y = -2x - x^2$; (-2, 4)

$m = \lim_{\Delta x \to 0} \frac{2(x+\Delta x) - (x+\Delta x)^2 - (2x - x^2)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{2x + 2\Delta x - x^2 - 2x\Delta x - \Delta x^2 - 2x + x^2}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{2\Delta x - 2x\Delta x - \Delta x^2}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\Delta x(2 - 2x - \Delta x)}{\Delta x}$

$\lim_{\Delta x \to 0} 2 - 2x - \Delta x$

$\lim_{\Delta x \to 0} 2 - 2x = 2 - 2.(-2) = 2 + 4 = 6$

R_T

$y - y^0 = m(x - x_0)$

$y - 4 = 6(x + 2)$

$y - 4 = 6x + 12$

$y - 6x - 4 - 12$

$-6x + y - 16 = 0$

$m_t \cdot m_n = -1$

$6 \cdot m_n = -1$

$m_n = -\frac{1}{6}$

$m_n = \frac{1}{6}$

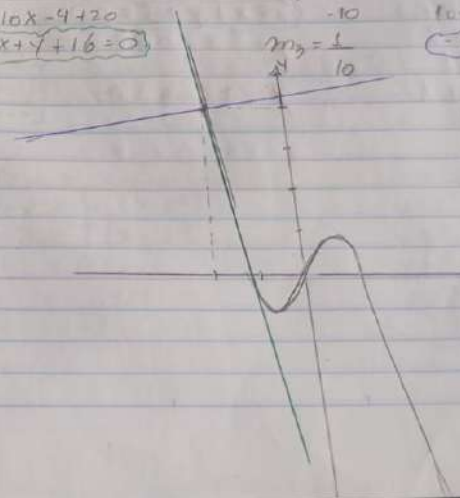
R_n

$y - y^0 = m(x - x^0)$

$y - 4 = \frac{1}{6}(x + 2)$

$6y - 24 = x + 2$

$-x + 6y - 26 = 0$



23) Ache uma equação da reta tangente à curva $y = 2x^2 + 3$ que é paralela à reta $8x - y + 3 = 0$

$m = \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 + 3 - (2x^2+3)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{2(x^2 + 2x\Delta x + \Delta x^2) + 3 - 2x^2 - 3}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{2x^2 + 4x\Delta x + 2\Delta x^2 + 3 - 2x^2 - 3}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{4x\Delta x + 2\Delta x^2}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\Delta x(4x + 2\Delta x)}{\Delta x}$

$\lim_{\Delta x \to 0} 4x + 2\Delta x$

$\lim_{\Delta x \to 0} 4x$

$m_1 = 4x \quad y = 2(2)^2 + 3$

$8 = 4x \quad y = 2 \cdot 4 + 3$

$x = \frac{8}{4} \quad y = 8 + 3$

$x = 2 \quad y = 11$

$(2, 11)$

$m_1 = m_2$

$8x - y + 3 = 0$

$y = 8x + 3$ (Coef. Angular)

reta Tangente

$y - y^0 = m(x - x^0)$

$y - 11 = 8(x - 2)$

$y - 11 = 8x - 16$

$-8x + y - 11 + 16 = 0$

$\boxed{-8x + y + 5 = 0}$

24) Ache uma equação da reta tangente à curva $y = x^3 - 3x$ que é perpendicular à reta $2x + 18y - 9 = 0$.

$m = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - 3(x+\Delta x) - (x^3 - 3x)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - 3x - 3\Delta x - x^3 + 3x}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - 3\Delta x}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\Delta x(3x^2 + 3x\Delta x + \Delta x^2 - 3)}{\Delta x}$

$\lim_{\Delta x \to 0} 3x^2 + 3x\Delta x + \Delta x^2 - 3$

$\lim_{\Delta x \to 0} 3x^2 - 3 \Rightarrow 3 \cdot 0^2 - 3 = 3$

$m = 3x^2 - 3 \quad y = x^3 - 3x$

$9 = 3x^2 - 3 \quad y = (-2)^3 - 3(-2)$

$12 = 3x^2 \quad y = -8 + 6$

$\frac{x^2 = 12}{3} \quad y = -2$

$x^2 = 4 \quad y = 2^3 - 3 \cdot 2 \quad (2, 2)$

$x = \sqrt{4} \quad y = 8 - 6 \quad (-2, -2)$

$x = \pm 2 \quad y = 2$

Para os pontos $(2, 2)$

$y - y^0 = m(x - x^0)$

$y - 2 = 9(x - 2)$

$y - 2 = 9x - 18$

$y - 9x - 2 + 18 = 0$

$\boxed{-9x + y + 16 = 0}$

$m_2 = -\frac{1}{9}$

$2x + 18y - 9 = 0$

$18y = -2x + 9$

$y = -\frac{2}{18}x + \frac{9}{18}$

$y = -\frac{1}{9}x + \frac{1}{2}$

$m_1 = -\frac{1}{9}$

$m_1 = 9$

Para os pontos $(-2, -2)$

$y - y^0 = m(x - x^0)$

$y + 2 = 9(x + 2)$

$y + 2 = 9x + 18$

$y - 9x + 2 - 18 = 0$

$\boxed{-9x + y - 16 = 0}$

Nos exercícios de 25 a 30, ache $f'(x)$ aplicando a fórmula (3)

25) $f(x) = 7x + 3$

$f'(x) = \lim_{\Delta x \to 0} \frac{7(x+\Delta x)+3-(7x+3)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{7x+7\Delta x+3-7x-3}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{7\Delta x}{\Delta x}$ $\quad f'(x) = 7$

$\lim_{\Delta x \to 0} 7$

26) $f(x) = 3x^2 - 2x + 1$

$f'(x) = \lim_{\Delta x \to 0} \frac{3(x+\Delta x)^2 - 2(x+\Delta x)+1-(3x^2-2x+1)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{3(x^2+2x\Delta x+\Delta x^2)-2x-2\Delta x+1-3x^2+2x-1}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{3x^2+6x\Delta x+3\Delta x^2-2x-2\Delta x+1-3x^2+2x-1}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{6x\Delta x+3\Delta x^2-2\Delta x}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\Delta x(6x+3\Delta x-2)}{\Delta x}$ $\quad f'(x) = 6x-2$

$\lim_{\Delta x \to 0} 6x-2$

Nos exercícios de 31 a 38, ache a derivada indicada:

31) $\frac{d}{dx}(8-x^3)$

$\frac{d}{dx} = \lim_{\Delta x \to 0} \frac{8-(x+\Delta x)^3-(8-x^3)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{8-(x^3+3x^2\Delta x+3x\Delta x^2+\Delta x^3)-8+x^3}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{8-x^3-3x^2\Delta x-3x\Delta x^2-\Delta x^3-8+x^3}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{-3x^2\Delta x-3x\Delta x^2-\Delta x^3}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\Delta x(-3x^2-3x\Delta x-\Delta x^2)}{\Delta x}$

$\lim_{\Delta x \to 0} -3x^2$ $\quad \frac{d}{dx} = -3x^2$

35) $\frac{d}{dx}\left(\frac{2x+3}{3x-2}\right)$ $\quad \frac{-13}{(3x-2)^2}$

$\frac{d}{dx} = \lim_{\Delta x \to 0} \frac{\frac{2(x+\Delta x)+3}{3(x+\Delta x)-2} - \frac{2x+3}{3x-2}}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\frac{2x+2\Delta x+3}{3x+3\Delta x-2} - \frac{3x-3}{3x+2}}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\frac{6x^2+6x\Delta x+9x+6\Delta x+6-6x^2-9x+6x+9\Delta x-6}{3x+3\Delta x-2-3x-2}}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{4\Delta x-9\Delta x}{3x+3\Delta x-2-3x-2} \div \Delta x$ $\quad \lim_{\Delta x \to 0} \frac{\Delta x(4-9)}{\Delta x(3)}$

35) $\frac{d}{dx}\left(\frac{2x+3}{3x-2}\right)$

$$\lim_{\Delta x\to 0}\frac{\frac{2(x+\Delta x)+3}{3(x+\Delta x)-2}-\frac{2x+3}{3x-2}}{\Delta x}$$

$$\lim_{\Delta x\to 0}\frac{6x^2+6x\Delta x+9x-4x-4\Delta x-6-6x^2-9x-6x\Delta x-9\Delta x+4x+6}{[(3x+3\Delta x)-2](3x-2)\,\Delta x}$$

$$\lim_{\Delta x\to 0}\frac{-4\Delta x-9\Delta x}{(3x+3\Delta x-2)(3x-2)\,\Delta x}$$

$$\lim_{\Delta x\to 0}\frac{\Delta x(-4-9)}{(3x-2)(3x-2)\,\Delta x}$$

$$\lim_{\Delta x\to 0}\Delta x(-4-9)\cdot\frac{1}{(3x-2)^2}\cdot\frac{1}{\Delta x}$$

$$\lim_{\Delta x\to 0}\frac{\Delta x(-4-9)}{\Delta x(3x-2)^2}$$

$$\lim_{\Delta x\to 0}\frac{-13}{(3x-2)^2}$$

$$\frac{d'}{dx}=\frac{-13}{(3x-2)^2}$$

36) $\frac{d}{dx}\left(\sqrt{3x+5}'\right)$

$$\lim_{\Delta x\to 0}\frac{\sqrt{3(x+\Delta x)+5}'-\sqrt{3x+5}'}{\Delta x}\cdot\frac{\sqrt{3(x+\Delta x)+5}'+\sqrt{3x+5}'}{\sqrt{3(x+\Delta x+5}'+\sqrt{3x+5}'}$$

$$\lim_{\Delta x\to 0}\frac{3x+3\Delta x+5-3x-5}{\Delta x\left(\sqrt{3(x+\Delta x+5}'+\sqrt{3x+5}'\right)}$$

$$\lim_{\Delta x\to 0}\frac{3\Delta x}{\Delta x\left(\sqrt{3(x+\Delta x+5)}+\sqrt{3x+5}'\right)}$$

$$\lim_{\Delta x\to 0}\frac{3}{\sqrt{3x+3\Delta x+5}'+\sqrt{3x+5}'}$$

$$\lim_{\Delta x\to 0}\frac{3}{\sqrt{3x+5}'+\sqrt{3x+5}'}=\frac{3}{2\sqrt{3x+5}'}$$

$$D_x'=\frac{3}{2\sqrt{3x+5}'}$$

Nos exercícios de 43 à 46, ache $f'(a)$ aplicando a fórmula (4)

43) $f(x)=4-x^2$; $a=5$

$$f'(5)=\lim_{\Delta x\to 0}\frac{4-(5+\Delta x)^2-(4-5^2)}{\Delta x}$$

$$\lim_{\Delta x\to 0}\frac{4-25-10\Delta x-\Delta x^2-4+25}{\Delta x}$$

$$\lim_{\Delta x\to 0}\frac{-10\Delta x-\Delta x^2}{\Delta x}$$

$$\lim_{\Delta x\to 0}\frac{\Delta x(-10+\Delta x)}{\Delta x}$$

$$\lim_{\Delta x\to 0}-10+\Delta x^{\to 0}=-10$$

$f'(x)=-10$

Usando a fórmula (7) $\quad \frac{f(x)-f(a)}{x-a}$

48) $f(x)=x^2-x+4$; $a=4$

$$f(4)=\lim_{x\to 4}\frac{x^2-x+4-(4^2-4+4)}{x-4}$$

$$\lim_{x\to 4}\frac{(x^2-x+4-16)}{(x-4)}$$

$$\lim_{x\to 4}\frac{(x^2-x-12)}{(x-4)}$$

$$\lim_{x\to 4}\frac{(x-4)(x+3)}{(x-4)}$$

$$\lim_{x\to 4}x+3=4+3=7$$

$f'(x)=7$

$x^2-x-12 \mid x-4$

$-x^2+4x \quad (x+3)$

$3x-12$

$-3x+12$

52) $y = \frac{4}{2x-5}$

$$\lim_{\Delta x \to 0} \frac{\frac{4^{-2}}{2(x+\Delta x)-5} - \frac{4^{-2}}{2x-5}}{\Delta x}$$

$$4 \lim_{\Delta x \to 0} \frac{(2x-5)-[2(x+\Delta x)-5]}{[2(x+\Delta x)-5](2x-5)} \cdot \frac{1}{\Delta x}$$

$$4 \lim_{\Delta x \to 0} \frac{2x-5-2x-2\Delta x+5}{[2(x+\Delta x)-5](2x-5)} \cdot \frac{1}{\Delta x}$$

$$4 \lim_{\Delta x \to 0} \frac{-2\Delta x}{(2x+2\Delta x-5)(2x-5)} \cdot \frac{1}{\Delta x}$$

$$4 \lim_{\Delta x \to 0} \frac{-2\Delta x}{\Delta x (2x+\Delta x-5)(2x-5)}$$

$$4 \lim_{\Delta x \to 0} \frac{-2}{(2x-5)(2x-5)}$$

$$\frac{d}{dx} = \frac{-8}{(2x-5)^2}$$

$$4 \lim_{\Delta x \to 0} \frac{-2}{(2x-5)^2} = \frac{-8}{(2x-5)^2}$$

57) $y = \sqrt[3]{x}$

$$\frac{d}{dx} = \lim_{\Delta x \to 0} \frac{\sqrt[3]{x+\Delta x} - \sqrt[3]{x}}{\Delta x} \cdot \frac{\sqrt[3]{x+\Delta x}^2 + \sqrt[3]{x}}{\sqrt[3]{x+\Delta x}^2 + \sqrt[3]{x}}$$

$$\lim_{\Delta x \to 0} \frac{(x+\Delta x - x)}{\Delta x (\sqrt[3]{x+\Delta x}^2 + \sqrt[3]{x})}$$

$$\lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x (\sqrt[3]{x+\Delta x}^2 + \sqrt[3]{x})}$$

$$\lim_{\Delta x \to 0} \frac{1}{\sqrt[3]{x}^2 + \sqrt[3]{x}}$$

$$\lim_{\Delta x \to 0} \frac{1}{\sqrt[3]{x^2}} = \frac{1}{x^{\frac{2}{3}}}$$

$$\frac{d}{dx} = \frac{1}{x^{\frac{2}{3}}}$$

Exercícios 3.2

Do 1 ao 20 | fazer 1 + cada 5
Do 21 ao 29 | fazer 21 + cada 2
fazer 29, 30, 31

1) $f(x) = \begin{cases} x+2 & \text{se } x \leq -4 \\ -x+6 & \text{se } x > -4 \end{cases}$ $x_1 = -4$

$f'(x_1) = \lim_{x \to x_1} \frac{f(x) - f(x_1)}{x - x_1}$

$y_1 = x+2$ (-3, 2)
$y_2 = -x+6$ (-6, -6)

(B) $f(-4) = x+2$
$f(-4) = -4+2$
$f(-4) = -2$
$\lim_{x \to -4^+} -x+6 = -(-4)+6 = 4+6 = 10$
$\lim_{x \to -4^-} x+2 = -4+2 = -2$

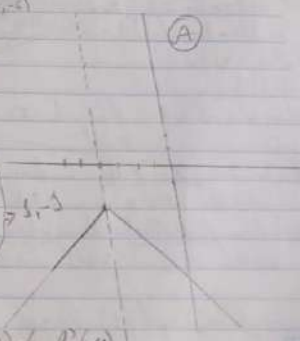
(C) $f'_-(-4) = \lim_{x \to -4^-} \frac{x+2-(-2)}{x-(-4)} \Rightarrow \lim_{x \to -4^-} \frac{x+4}{x+4} = 1$

$f'_+(-4) = \lim_{x \to -4^+} \frac{-x-6-(-2)}{x-(-4)} \Rightarrow \lim_{x \to -4^+} \frac{-x-6+2}{x+4}$

$\lim_{x \to -4^+} \frac{-(x+4)}{x+4} = -1$

(D) não é derivável pois $f'_-(-4) \neq f'_+(-4)$

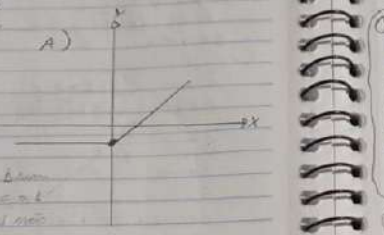
(A)



14) $f(x) = \begin{cases} -1 & \text{se } x < 0 \\ x-1 & \text{se } x \geq 0 \end{cases}$ $x_1 = 0$

B) $f(x) = x-1$
$f(0) = 0-1$
$f(0) = -1$
$\lim_{x \to 0} 1 = -1$
$\lim_{x \to 0} x-1 = 0-1 = -1$

A)



C) $f'(0) = \lim_{x \to x_1} \frac{f(x)-f(x_1)}{x-x_1}$

$\lim_{x \to 0^-} \frac{-1-(-1)}{x-0} = \frac{-1+1}{x} = \frac{0}{x} = 0$

$f'_+(0) = \lim_{x \to x_1^+} \frac{f(x)-f(x_1)}{x-x_1} = \lim_{x \to 0^+} \frac{(x-1)-(-1)}{x-0} = \frac{x-1+1}{x-0} = \frac{1}{1} = 1$

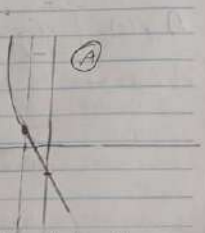
D) Não é derivada pois $f'_-(0) \neq f'_+(0)$.

16) $f(x) = \begin{cases} x^2 & \text{se } x < -1 \\ -1-2x & \text{se } x \geq -1 \end{cases}$ $x_1 = -1$

B) $f(-1) = -1-2x$
$f(-1) = -1-2(-1)$
$f(-1) = -1+2$
$f(-1) = 1$

$\lim_{x \to -1^-} x^2 = -1^2 = 1$ $\lim_{x \to -1^+} -1-2x = -1-2(-1) = 1$

A)



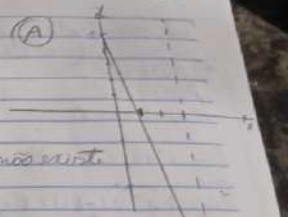
C) $f'(x_1) = \lim_{x \to x_1} \frac{f(x)-f(x_1)}{x-x_1} = \lim_{x \to -1^-} \frac{x^2-1}{x-(-1)} = \lim_{x \to -1^-} \frac{(x+1)(x-1)}{x+1} = -1-1 = -2$

$f'_+(x_1) = \lim_{x \to x_1} \frac{f(x)-f(x_1)}{x-x_1} = \lim_{x \to -1^+} \frac{(-1-2x)-(-1)}{x-(-1)} = \lim_{x \to -1^+} \frac{-1-2x+1}{x+1} = \frac{-2(x+1)}{(x+1)} = -2$

D) como $f'(-1) = f(-1)$ logo $f'(-1) = -2$

15) $f(x) = \begin{cases} 5-6x & \text{se } x \leq 3 \\ -1-x^2 & \text{se } x > 3 \end{cases}$ $x_1 = 3$

A)



B) $f(3) = 5-6x$
$f(3) = 5-6 \cdot 3$
$f(3) = 5-18$
$f(3) = -13$

$\lim_{x \to 3^-} 5-6x = 5-6 \cdot 3 = -13$ $\lim_{x \to 3^+} -4-x^2 = -4-3^2 = -4-9 = 5$

$\lim f(x)$ não existe

C) $f'(x_1) = \lim_{x \to x_1} \frac{f(x)-f(x_1)}{x-x_1}$

$= \lim_{x \to 3} \frac{(5-6x)-(-13)}{(x-3)} = \lim_{x \to 3^-} \frac{-6x+5+13}{(x-3)}$

$\lim_{x \to 3^-} \frac{-6x+18}{(x-3)} = \frac{-6(x-3)}{(x-3)} = -6$

$f'_+(x_1) = \lim_{x \to x_1} \frac{f(x)-f(x_1)}{x-x_1} = \lim_{x \to 3^+} \frac{(-4-x^2)-(-13)}{(x-3)}$

$= \lim_{x \to 3^+} \frac{-x^2-1+13}{(x-3)} = \lim_{x \to 3^+} \frac{-x^2+9}{x-3} = \lim_{x \to 3^+} \frac{(x-3)(x+3)}{(x-3)} = -3-3 = -6$

D) como $f'_-(x_1) = f'_+(x_1)$ logo $f'(x_1) = -6$.

a) $f(x) = \begin{cases} x^2+1 & x \neq -1 \\ 1-x^2 & x \geq -1 \end{cases}$ $x_1 = -1$

(B) $f(x) = 1-x^2$

$f(-1) = 1-(-1)^2$

$f(-1) = 1-1 = 0$

$\lim_{x \to -1} = 0$

$\lim_{x \to -1^+} x^2+1 = 1+1 = 2$

$\lim_{x \to -1^-} 1-x^2 = 1-(-1)^2 = 1-1 = 0$

$\lim_{x \to -1} f(x)$ não existe

não é continua

(C) $f'(x_1) = \lim_{x \to x_1} \frac{f(x)-f(x_1)}{x-x_1} = \lim_{x \to -1^-} \frac{(x^2+1)-2}{x+1} = \lim_{x \to -1^-} \frac{x^2-1}{x+1}$

$\lim_{x \to -1^-} \frac{(x-1)(x-1)}{x+1} = -1-1 = -2$

$f_+'(x_1) = \lim_{x \to x_1} \frac{f(x)-f(x_1)}{x-x_1} = \lim_{x \to -1^+} \frac{(1-x^2)-0}{x+1} = \lim_{x \to -1^+} \frac{-x^2+1}{x+1}$

$\lim_{x \to -1^+} \frac{-(x-1)}{x+1}$ $\lim_{x \to -1^+} \frac{-(x-1)(x+1)}{x+1} = \lim_{x \to -1^+} (-x+1) = 2$

(D) como $f_-'(x_1) \neq f_+'(x_1)$ então, $f'(x_1)$ = não existe

22) Dado $f(x) = \sqrt{4-x^2}$, (a) prove que f é contínua no intervalo $[-2, 2]$

(a) $f(2) = \sqrt{4-2^2}$ $f(-2) = \sqrt{4+2^2}$

$f(2) = 2-2 = 0$ $f(-2) = \sqrt{4+4} = 4$

$\lim_{x \to 2^-} \sqrt{4-x^2} = \lim_{x \to 2^-} 2-x = 2-2 = 0$

$\lim_{x \to -2^-} \sqrt{4-x^2} = \lim_{x \to -2^-} 2-x = 2+2 = 4$

(B) prove que nem $f_+'(2)$ e nem $f_-'(2)$ existem.

$f_+'(2)$ $\lim_{x \to 2^+} \frac{\sqrt{4-x^2}-0}{x+2} = \lim_{x \to 2^+} \frac{\sqrt{(2-x)(2+x)}}{\sqrt{(x+2)^2}} = \lim_{x \to 2^+} \frac{\sqrt{(2-x)(2+x)}}{\sqrt{(x+2)}\sqrt{(x+2)}}$

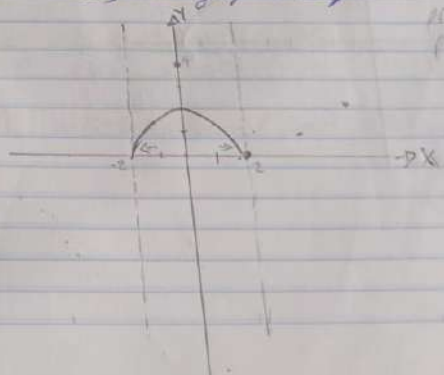
$\lim_{x \to -2^+} \frac{\sqrt{2-x}}{\sqrt{x+2}} = \frac{\sqrt{2+2}}{\sqrt{-2+2}} = \frac{2}{0} = +\infty$

$f_-'(2) = \lim_{x \to 2^-} \frac{\sqrt{4-x^2}-0}{x-2} = \lim_{x \to 2^-} \frac{\sqrt{(2-x)(2+x)}}{-(2-x)} = -\lim_{x \to 2^-} \frac{\sqrt{(2-x)(2+x)}}{(2-x)}$

$-\lim_{x \to 2^-} \frac{\sqrt{(2+x)(2+x)}}{\sqrt{(2-x)(2-x)}} = -\lim_{x \to 2^-} \frac{\sqrt{2+x}}{\sqrt{2-x}} = \frac{2}{0} = \infty$

$f(x)$ não é derivável

c) faça um esboço do gráfico de f.



$f(0) = \sqrt{4-0^2}$

$f(0) = 2$

2d) Seja $f(x) = \sqrt{8-x}$.

a) Prove que f é contínua à esquerda de 8.

$f(8) = \sqrt{8-8} = 0$

$\lim_{x \to 8^-} \sqrt{8-x} = \sqrt{8-8} = 0$

$f(8) = \lim_{x \to 8^-} f(x)$

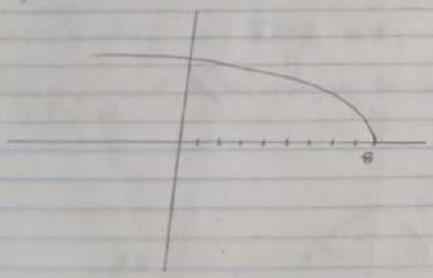
f é contínua à esquerda em $x = 8$

b) Prove que $f'(8)$ não existe.

$f'(8) = \lim_{x \to 8} \frac{f(x) - f(8)}{x - 8} = \lim_{x \to 8^-} \frac{\sqrt{8-x} - 0}{x - 8}$

$= \lim_{x \to 8^-} \frac{\sqrt{8-x}}{-\sqrt{(8-x)^2}} = -\lim_{x \to 8^-} \frac{\sqrt{8-x}}{\sqrt{(8-x)(8-x)}} = -\frac{1}{\lim_{x \to 8^-} \sqrt{8-x}}$ $\quad \frac{1}{0^+} = -\infty$

c) Faça um esboço do gráfico de f.



2g) Dada $f(x) = \begin{cases} x^2 - 7 & \text{se } 0 < x < b \\ 6 & \text{se } x \geq b \\ x & \end{cases}$

a) Determine um valor de b para que f seja contínua em b.

$\lim_{x \to b^-} x^2 - 7 = \lim_{x \to b^-} b^2 - 7 = b^2 - 7$

$\lim_{x \to b^+} 6 = 6$ $\quad$ $\lim_{x \to b^+} x = b$

$b^2 - 7 = 6$

$b^2 - 7b - 6 = 0$

$b = 3$

b) f é derivável em b, encontrado na parte a?

$f'(3) = \lim_{x \to 3^-} \frac{x^2 - 7 - 2}{x - 3} = \lim_{x \to 3^-} \frac{x^2 - 9}{x - 3}$

$= \lim_{x \to 3^-} \frac{(x-3)(x+3)}{(x-3)} = \lim_{x \to 3^-} x + 3 = 3 + 3 = 6$

$f'_+(3) = \lim_{x \to 3^+} \frac{\frac{6}{x} - 2}{x - 3} = \lim_{x \to 3^+} \frac{\frac{6 - 2x}{x}}{x - 3}$

$\lim_{x \to 3^+} \frac{6 + 2x}{x(x-3)}$ $\quad$ $\lim_{x \to 3^+} \frac{-2(x-3)}{x(x-3)} = -\frac{2}{3}$

Logo $f'(3)$ não é derivável pois $f'_-(3) \neq f'_+(3)$

32) Ache os valores de a e b, tais que f seja derivável em 1 se $f(x) = \begin{cases} x^2 & \text{se } x < 1 \\ ax + b & \text{se } x \geq 1 \end{cases}$

$f'(1) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - x_1} = \lim_{x \to 1^-} \frac{(x^2 - 1)}{(x-1)} \lim_{x \to 1^-} \frac{(x-1)(x+1)}{(x-1)} = 1 + 1 = 2$

$f'(1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - x_1} = \lim_{x \to 1^+} \frac{(ax + b - (a+b))}{(x-1)} \lim_{x \to 1^+} \frac{ax + b - a - b}{(x-1)}$

$\lim_{x \to 1^+} \frac{a(x-1)}{(x-1)} \lim_{x \to 1^+} a = a$

como $f'(1) = f'(1)$ efetuamos que $\boxed{a = 2}$

$f(1) = a \cdot x + b$

$f(1) = 2 \cdot 1 + b$

$f(1) = 2 + b$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 = 1$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} ax + b = \lim_{x \to 1^+} 2x + b = 2 + b$

se $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} \Rightarrow 2 + b = 1$

$b = 1 - 2$

$\boxed{b = -1}$

33) ache os valores de a e b, tais que f seja derivável em 2 se $f(x) = \begin{cases} ax + b & \text{se } x \leq 2 \\ 2x^2 - 1 & \text{se } 2 < x \end{cases}$

$f'(2) = \lim_{x \to 2^-} \frac{f(x) - f(x_1)}{x - x_1} = \lim_{x \to 2^-} \frac{ax + b - (2a + b)}{(x - 2)}$

$\lim_{x \to 2^-} \frac{ax + b - 2a - b}{(x-2)} \lim_{x \to 2^-} \frac{a(x-2)}{(x-2)} \lim_{x \to 2^-} a = a$

$f'(2) = \lim_{x \to 2^+} \frac{f(x) - f(x_1)}{(x - x_1)} = \lim_{x \to 2^+} \frac{2x^2 - 1 - 7}{(x-2)} \cdot \lim_{x \to 2^+} \frac{2x^2 - 8}{x - 2}$

$\lim_{x \to 2^+} \frac{2(x^2 - 4)}{(x-2)} \cdot \lim_{x \to 2^+} \frac{2(x-2)(x+2)}{(x-2)} \cdot \lim_{x \to 2^+} 2(x+2)$

$= \lim_{x \to 2^+} 2(2+2) = 8$

se $f'(2) = f'(2)$ logo $\boxed{a = 8}$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 2x^2 - 1 = 2 \cdot 2^2 - 1 = 2 \cdot 4 - 1 = 7$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} ax + b = 8 \cdot 2 + b = 16 + b$

se $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} \Rightarrow 16 + b = 7$

$b = 7 - 16$

$\boxed{b = -9}$

Sessão 3.3 do 1 ao 45 fazer 1 a cada 3

Nos exercícios de 1 ao 24 derivar a função dada, aplicando os teoremas dessa seção.

2) $g(x) = 8 - 3x$ $= D_x(8-3x)$
$= D_x 8 - D_x[3x]$
$= 0 - 3x^0$
$D_x[g(x)] = -3$

4) $f(x) = 4x^2 + x + 1$ $D_x[f(x)] = 8x + 1$

7) $f(x) = \frac{1}{8}x^3 - x^4$ $= D_x[f(x)] = 8 \cdot \frac{1}{8}x^2 - 4x^3$
$D_x = x^2 - 4x^3$

11) $V(r) = \frac{4}{3}\pi r^3$ $D_x[V(r)] = 3 \cdot \frac{4}{3}\pi r^2$
$D_x = 4\pi r^2$

14) $f(x) = \frac{x^3}{3} + \frac{3}{x^3}$ $= f(x)\ \frac{1}{3}x^3 + 3\frac{x^{-3}}{1}$
$D_x[f(x)] = \frac{1}{3}x^3 + 3x^{-3}$
$D_x = \frac{1}{3}x^3 + 3D_x - 3x^{-3-1}$
$D_x = \frac{1}{3}\cdot 3x^2 + 3 \cdot -3x^{-4}$
$D_x = x^2 - 9x^{-4}$
$D_x = x^2 - \frac{9}{x^4}$

16) $f(x) = x^4 - 5 + x^{-2} + 4x^{-4}$
$D_x[f(x)] = x^3 - 5 + x^{-2} + 4x^{-5}$
$D_x = 9x^3 - 2x^{-3} - 16x^{-5}$

18) $H(x) = \frac{5}{6x^5}$ $D_x[H(x)] = D_x\frac{5}{6}x^{-5}$
$= \frac{5}{6} D_x - 5x^{-6}$
$= \frac{25}{6}x^{-6}$
$= \frac{25}{6x^6}$

20) $g(x) = (2x^2+5)(4x-1)$ $D_x[g(x)] = (4x)\cdot(\ldots) + (2x^2+5)\cdot(\ldots)$
$= 16x^2 - 4x + 8x^2 + \ldots$
$= 24x^2 - 4x + 20$

22) $f(x) = (4x^2+3)^2 = (4x^2+3)\cdot(4x^2+3)$
$D_x[f(x)] = 8x\cdot(4x^2+3) + (4x^2+3)\cdot 8x$
$= 32x^3 + 24x + 32x^3 + 24x$
$= 64x^3 + 48x$

Nos exercícios de 25 a 36, calcule a derivada indicada, aplicando os teoremas deste capítulo.

25) $D_x [(x^2-3x+2)(2x^3+1)]$

$= (2x-3)\cdot(2x^3+1) + (x^2-3x+2)\cdot(6x^2)$

$= 4x^4+2x-6x^2-3+6x^4-18x^3+12x^2$

$= 10x^4-24x^3+12x^2+2x-3$,,

29) $\frac{d}{dx}\left(\frac{x^2+2x+1}{x^2-2x+1}\right)$ $\frac{f(x)}{g(x)}$ $\frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2}$

$= \frac{(2x+2)(x^2-2x+1) - (x^2+2x+1)(2x-2)}{(x^2-2x+1)^2}$

$= \frac{2x^3-4x^2+2x+2x^2-4x+2 - 2x^3-2x^2+4x^2-4x+2x-2}{((x-1)^2)^2}$

$= \frac{(2x^3-2x^2-2x+2) - (2x^3+2x^2-2x-2)}{(x-1)^4}$

$= \frac{-4x^2+4}{(x-1)^4}$

$= \frac{4(x^2+1)}{(x-1)^4} = \frac{4(x+1)(x-1)}{(x-1)^4} = \frac{4(x+1)}{(x-1)^3}$,,

30) $\frac{d}{dx}\left(\frac{x^4-2x^2+5x+1}{x^4}\right)$ $\frac{f(x)}{g(x)}$

$\frac{d}{dx} = \frac{(4x^3-4x+5)(x^4) - (x^4-2x^2+5x+1)(4x^3)}{(x^4)^2}$

$\frac{d}{dx} = \frac{(4x^7-4x^5+5x^4) - (4x^7-8x^5+20x^4+4x^3)}{x^8}$

$\frac{d}{dx} = \frac{4x^5-15x^4-4x^3}{x^8} = \frac{4x^3-15x-4}{x^5}$

38) Ache uma equação da reta normal à curva $y = 4x^2-8x$ no ponto $(1,-4)$.

$m = \frac{dy}{dx}\ 8x-8$ no ponto $x=1$

$8\cdot 1-8 = 0$ → horizontal

m_n = não existe

40) ache uma equação da reta ~~tangente~~ normal à curva $y = 8/(x^2+4)$ no ponto $(2,1)$.

$y = 8/(x^2+4)$

$m = \frac{dy}{dx}\ \frac{8}{(x^2+4)} = \frac{0\cdot(x^2+4) - 8(2x)}{(x^2+4)^2} = \frac{-16x}{(x^2+4)^2}$

$y-y' = m(x-x')$

$y-1 = -1(x-2)$

$2y-2 = -x+2$

$x+2y-4=0$

Em $x=2$

$\frac{-16\cdot 2}{(2^2+4)^2} = \frac{-32}{8^2} = -\frac{32}{64} = -\frac{1}{2}$

45) Ache uma equação da reta, uma das retas que passam pelo ponto (4,13), que sejam tangentes à curva $y = 2x^2 - 1$.

$m = \frac{dy}{dx} = 2x^2 - 1 = 4x$

Para $x = 1$ $\quad m = 4.1 = 4$

$y - 13 = 4(x - 4)$

$y - 13 = 4x - 16$

$-4x + y + 3 = 0$ ou $\boxed{y = 4x - 3}$

para $x = 7$ $\quad m = 4.7 = 28$

$y - 13 = 28(x - 4)$

$y - 13 = 28x - 112$

$-28x + y + 99 = 0$ ou $\boxed{y = 28x - 99}$

$y - y' = m(x - x')$

$13 - (2x^2 - 1) = 4x(4 - x)$

$13 - 2x^2 + 1 = 4x^2 + 16x$

$-2x^2 + 14 + 4x^2 - 16x = 0$

$2x^2 - 16x + 14 = 0$

$x^2 - 8x + 7 = 0$

$\Delta = 64 - 4.1.7$

$\Delta = 64 - 28$

$\Delta = 36$

$x = \frac{+8 \pm 6}{2}$

$x' = \frac{8 + 6}{2} = 7$

$x'' = \frac{8 - 6}{2} = \frac{2}{2} = 1$

Seção 3.5 Fazer: 1 ao 2, 06, 07, 11, 13, 15 e do 17 ao 30; Fazer 1a e de 3 do 31 ao 42; 1 ( ) 1a cada 3 do 51 ao 54; 1 ( ) 1a cada 2

1) Provar $D_x(\cotg x) = -\operatorname{cossec}^2 x$

$D_x = \frac{\cos x}{\sen x} = \frac{(-\sen x)(\sen x) - (\cos x)(\cos x)}{\sen^2 x}$

$= \frac{(-\sen^2 x) - (\cos^2 x)}{\sen^2 x}$

$= -\frac{(\sen^2 x) + (\cos^2 x)}{\sen^2 x} =$

$= \frac{-(1)}{\sen^2 x} = \frac{-1}{\sen^2 x} = -\operatorname{cossec}^2 x$

06) $f(x) = 4 \sec x - 2 \operatorname{cossec} x$

$D_x = 4[(\sec x).(\tg x)] - 2[-\operatorname{cossec}(\cotg x)]$

$D_x = 4 \sec x \tg x + 2 \operatorname{cossec} x \cotg^2 x$

07) $f(x) = 2T \cos T$

$D_x = 2T(\cos T) + 2T(-\sen T)$

$D_x = 2 \cos T - 2T \sen T$

$D_x = 2(\cos T - T \sen T)$

11) $h(x) = 4 \cdot \operatorname{sen} x \cdot \cos x$

$4\,Dx = (\cos x)(\cos x) + (\operatorname{sen} x)(-\operatorname{sen} x)$

$Dx = 4(\cos^2 x - \operatorname{sen}^2 x)$

$\cos 2x \rightarrow \cos(2a) = \cos(a+a)$

$\cos 2a = \cos^2(a) - \operatorname{sen}^2(a)$

$Dx = 4 \cos 2x$

13) $f(x) = x^2 \cdot \cos x - 2x \operatorname{sen} x - 2\cos x$

$Dx = [x^2(-\operatorname{sen} x) + 2x \cos x] - [2x \cos x + 2 \operatorname{sen} x] + 2 \operatorname{sen} x$

$Dx = -x^2 \operatorname{sen} x$ //

$Dx = x^2 \cos x$

$= 2x \cdot (\cos x) + [(x^2)(-\operatorname{sen} x)]$

$[= x^2(-\operatorname{sen} x) + 2x \cos x]$

$Dx = -2x \operatorname{sen} x$

$Dx = -2(\operatorname{sen} x) + [-2x(\cos x)]$

$= -2x \cos x + 2 \operatorname{sen} x$

$Dx = -2\cos x$

$= -2(+\operatorname{sen} x)$

$= 2 \operatorname{sen} x$

15) $f(x) = 3 \sec x \operatorname{tg} x$

$Dx = 3(\sec x \operatorname{tg} x)(\operatorname{tg} x) + (\sec x)(\sec^2 x)$

$= 3(\sec x \operatorname{tg} x)$

$Dx = [3(\sec x \operatorname{tg} x) + (\sec^2 x)]$ colocando em evidência

$Dx = \operatorname{tg}^2 x = \sec^2 x$

$Dx = 3 \sec x (\operatorname{tg}^2 x + \sec^2 x)$

Calcule a derivada indicada.

17) $Dy(\operatorname{cotg} y \cdot \operatorname{cossec} y)$

$Dy(\operatorname{cotg} y \cdot \operatorname{cossec} y) = Dy(\operatorname{cotg} y) \cdot \operatorname{cossec} y + \operatorname{cotg} y \cdot Dy(\operatorname{cossec} y)$

$-\operatorname{cossec}^2 x \cdot \operatorname{cossec} y + \operatorname{cotg} y (-\operatorname{cossec} y \cdot \operatorname{cotg} y)$

$\operatorname{cossec}^2 y = 1 + \operatorname{cot}^2 y$

$= -\operatorname{cossec}^2 y \cdot \operatorname{cossec} y - \operatorname{cotg}^2 y \cdot \operatorname{cossec} y$

$-\operatorname{cossec} y (\operatorname{cossec}^2 y + \operatorname{cotg}^2 y)$

$-\operatorname{cossec} y (1 + \operatorname{cot}^2 y) + \operatorname{cotg}^2 y$

$-\operatorname{cossec} y (2 \operatorname{cot}^2 y + 1)$ //

21) $\frac{d}{dx}\left(\frac{\operatorname{sen} x}{1 - \cos x}\right) = \frac{Dx(\operatorname{sen} x) \cdot 1 - \cos x - \operatorname{sen} x \, Dx(1 - \cos x)}{(1 - \cos x)^2}$

$Dx(\operatorname{sen} x) = \cos x$

$Dx(1 - \cos x)$

$\operatorname{sen}^2 x + \cos^2 y = 1$

$\cos^2 x - 1 = \operatorname{sen}^2 x$

$\frac{d}{dx} = \frac{\cos \cdot (1 - \cos x) - \operatorname{sen} x (+\operatorname{sen} x)}{(1 - \cos x)^2}$

$\frac{d}{dx} = \frac{\cos - (\cos^2 x + \operatorname{sen}^2 x)}{(1 - \cos x)^2}$

$\frac{d}{dx} = \frac{\cos - 1}{(1 - \cos x)^2}$

$\frac{d}{dx} = \frac{1}{1 - \cos x}$

25) $\frac{d}{dy}\left(\frac{1+\operatorname{sen} y}{1-\operatorname{sen} y}\right) \cdot \frac{d}{dy} = \frac{0(1-\operatorname{sen} y) - (1+\operatorname{sen} y)(0-\cos y)}{(1-\operatorname{sen} y)^2}$

$\frac{d}{dy} = \frac{\cos y \cdot (1-\operatorname{sen} y) - (1+\operatorname{sen} y)(-\cos y)}{(1-\operatorname{sen} y)^2}$

$\frac{d}{dy} = \frac{\cos y - \cos y \operatorname{sen} y + \cos y + \operatorname{sen} y \cos y}{(1-\operatorname{sen} y)^2}$

$\frac{d}{dy} = \frac{2\cos y}{(1-\operatorname{sen} y)^2}$

27) $D_x[(x-\operatorname{sen} x)(x+\cos x)]$

$D_x = D_x(x-\operatorname{sen} x)\cdot(x+\cos x) + (x-\operatorname{sen} x)\cdot D_x(x+\cos x)$

$D_x = (1-\cos x)(x+\cos x) + (x-\operatorname{sen} x)\cdot(1-\operatorname{sen} x)$

nos exercícios de 31 a 42, ache $f'(a)$ para o valor de $f(a)$ do 31 ao 42 ⇒ 1 a cada 3

31) $f(x) = x \cdot \cos x \;;\; a = 0$

$f'(x) = D_x x \cdot \cos x + x \cdot D_x \cos x$

$f'(x) = 1 \cdot \cos x + x \cdot -\operatorname{sen} x$

$f'(x) = \cos x - x \operatorname{sen} x$

$f'(0) = \cos 0 - 0 \operatorname{sen} 0$

$f'(0) = 1 - 0.0$

$f'(0) = 1$

35) $f(x) = x^2 \cdot \operatorname{tg} x \;;\; a = \pi$

$f'(x) = D_x(x^2)\cdot \operatorname{tg} x + x^2 \cdot D_x(\operatorname{tg} x)$

$f'(x) = 2x \cdot \operatorname{tg} x + x^2 \cdot \sec^2 x$

$f'(x) = 2x \operatorname{tg} x + x^2 \sec^2 x$

$f'(x) = 2\pi \operatorname{tg} \pi + \pi^2 \sec^2 \pi$

$f'(x) = 2\pi \cdot 0 + \pi^2 \cdot (-1)^2$

$f'(x) = 0 + \pi^2 \cdot 1$

$f'(x) = \pi^2$

39) $f(x) = \underbrace{x \cos x}_{u} + \underbrace{x \operatorname{sen} x}_{v} \;;\; a = \frac{1}{4}\pi$

$f'(x) = D_x(x\cos x) + D_x(x \operatorname{sen} x)$

$f'(x) = 1 \cdot \cos x - x \operatorname{sen} x + 1 \cdot \operatorname{sen} x + x \cos x$

$f'(x) = \cos x - x \operatorname{sen} x + \operatorname{sen} x + x \cos x$

$f'(x) = \cos \frac{1}{4}\pi - \frac{1}{4}\pi \operatorname{sen} \frac{1}{4}\pi + \operatorname{sen} \frac{1}{4}\pi + \frac{1}{4}\pi \cos \frac{1}{4}\pi$

$f'(x) = \frac{\sqrt{2}}{2} - \frac{\pi}{4}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\pi}{4}\cdot\frac{\sqrt{2}}{2}$

$f'(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$f'(x) = \frac{2\sqrt{2}}{2}$

$f'(x) = \sqrt{2}$.

51) Ache uma equação da reta tangente ao gráfico da função ~~cosseno~~ no ponto

a) $x = 0$

b) $x = \frac{\pi}{3}$

c) $x = \pi$

a) $y - y^0 = m(x - x^0)$

$y - 0 = 1(x - 0)$

$y = x$

$y - x = 0$ $(0,0)$

$f(x) = \cos x$

$f'(0) = \cos 0 = 1$

$f(x) = x$

$f'(a) = 0$ $y = 0$

b) $f(x) = \operatorname{sen} x$

$f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$

$f'\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3}$

$f'\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$m = \frac{1}{2}$

$y - y^0 = m(x - x^0)$

$y - \frac{\sqrt{3}}{2} = \frac{1}{2}\left(x - \frac{\pi}{3}\right)$

$2y - \frac{2\sqrt{3}}{2} = x - \frac{\pi}{3}$

$2y - \sqrt{3} - x + \frac{\pi}{3} = 0$

$-x + 2y - \sqrt{3} + \frac{\pi}{3} = 0$

c) $f(x) = \operatorname{sen} x$

$f(\pi) = 0$

$(\pi, 0)$

$y - y^0 = m(x - x^0)$

$y - 0 = -1(x - \pi)$

$y = -x + \pi$

$y + x - \pi = 0$

$f'(x) = \cos x$

$f'(\pi) = -\cos\pi$

$f'\pi = -1$

$m = -1$

53) Ache uma equação da reta tangente ao gráfico da função tangente no ponto a) $x = 0$ b) $x = \frac{\pi}{4}$ c) $x = -\frac{\pi}{4}$

a) $f(x) = \operatorname{tg} x$

$f(0) = 0$ $(0,0)$

$f'(x) = \operatorname{tg} x$

$f'(x) = \sec^2 x$

$f'(0) = \frac{1}{(\cos^2 x)} = \frac{1}{1} = 1$

$m = 1$

$y - y^0 = m(x - x^0)$

$y - 0 = 1(x - 0)$

$y = x$

$y - x = 0$

b) $f(x) = \operatorname{tg} x$

$f\left(\frac{\pi}{4}\right) = 1$

$\left(\frac{\pi}{4}, 1\right)$

$f'(x) = \frac{1}{\cos^2 x}$

$f'\left(\frac{\pi}{4}\right) = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{1}{\frac{2}{4}} = \frac{4}{2} = 2$

$m = 2$

$y - y^0 = m(x - x^0)$

$y - 1 = 2\left(x - \frac{\pi}{4}\right)$

$y - 1 = 2x - \frac{2\pi}{4}$

$y - 1 = 2x - \frac{\pi}{2}$

$y - 2x + \pi - 1 = 0$

c) $f(x) = \operatorname{tg} x$ $\left(-\frac{\pi}{4}, -1\right)$

$f\left(-\frac{\pi}{4}\right) = -1$

$f'(x) = \frac{1}{\cos^2 x}$

$f'(x) = \frac{1}{\left(-\frac{\sqrt{2}}{2}\right)^2} = \frac{1}{\frac{2}{4}} = \frac{4}{2} = 2$

$y - y^0 = m(x - x^0)$

$y + 1 = 2\left(x + \frac{\pi}{4}\right)$

$y + 1 = 2x + \frac{2\pi}{4}$

$y + 1 = 2x + \frac{\pi}{2}$

$y + 2x + 1 + \frac{\pi}{2} = 0$

3,6

~~de 01 a 36~~ ; ~~fazer~~ 1 a cada 4
~~fazer 37 ao 38~~
~~fazer 44, 46, 47~~ $f'(g(x))$

1) $f(x) = (2x+1)^3$ (reg) $f(x) = x^3$ $g(x) = 2x+1$
$f'(x) = 3x^2$ $g'(x) = 2$

$f'(x) = 3(2x+1)^2 \cdot 2$
$f'(x) = 6(2x+1)^2$ ou $f \cdot g \cdot g'$

5) $f(t) = (2t^4 - 7t^3 + 2t - 1)^2$

$f'(t) = 2(2t^4 - 7t^3 + 2t - 1) \cdot (8t^3 - 21t^2 + 2)$

9) $f(x) = 4\cos 3x - 3\operatorname{sen} 4x$
$f'(x) = 4(-3\operatorname{sen} 3x) - 3(4\cos 4x)$
$f'(x) = -12\operatorname{sen} 3x - 12\cos 4x$
$f'(x) = -12(\operatorname{sen} 3x + \cos 4x)$

$4\cos 3x$
$4(-3\operatorname{sen} 3x)$
$-12\operatorname{sen} 3x$

$-3\operatorname{sen} 4x$
$-3(\cos 4x) \cdot 4$
$-3(4\cos 4x)$
$-12\cos 4x$

12) $f(x) = \cos(3x^2+1)$
$f'(x) = -\operatorname{sen}(3x^2+1) \cdot 6x$
$f'(x) = -6x\operatorname{sen}(3x^2+1)$

15) $\frac{d}{dt}(\operatorname{cotg}^4 t - \operatorname{cosec}^4 t)$

$\frac{d}{dt} = 4(\operatorname{cotg} t)^3 \cdot (-\operatorname{cosec}^2 t) - 4(\operatorname{cosec} t)^3 \cdot (-\operatorname{cosec} t \cdot \operatorname{cotg} t)$
$\frac{d}{dt} = 4(\operatorname{cotg} t)^3 \cdot (-\operatorname{cosec}^2 t) + 4\operatorname{cosec}^4 t \cdot \operatorname{cotg} t$
$\frac{d}{dt} = -4(\operatorname{cotg}^3 t \operatorname{cosec}^2 t) + 4(\operatorname{cosec}^4 t \operatorname{cotg} t)$
$\frac{d}{dt} = 4\operatorname{cotg} t \operatorname{cosec}^2 t(-\operatorname{cotg}^2 t + \operatorname{cosec}^2 t)$
$\frac{d}{dt} = 4\operatorname{cotg} t \operatorname{cosec}^2 t \cdot 1$

$-\operatorname{cot}^2 T + \operatorname{cosec}^2 t = 1$

$\frac{d}{dt} = 4\operatorname{cotg} t \operatorname{cosec}^2 t$ //

19) $D_x\{(2x-5)^{-1}(4x+3)^{-2}\}$
$D_x = (-1)(2x-5)^{-2} \cdot 2 \cdot (-2)(4x+3)^{-3} \cdot 4$
$D_x = -2(2x-5)^{-2} \cdot -8(4x+3)^{-3}$
$D_x = -2(2x-5)^{-2} \cdot (4x+3)^{-3} + (2x-5)^{-1} \cdot (-3)(4x+3)^{-4}$
$D_x = -2(2x-5)^{-2}(4x+3)^{-3} - 8(2x-5)^{-1}(4x+3)^{-4}$
$D_x = -2(2x-5)^{-2}(4x+3)^{-3}\{(4x+3) + 4(2x-5)\}$
$D_x = -2(2x-5)^{-2}(4x+3)^{-3}\{4x+3+8x-20\}$
$D_x = -2(2x-5)^{-2}(4x+3)^{-3}(12x-17)$

25) $f(x) = \left(\frac{2x-1}{3x^2+x-2}\right)^3$

$3\left(\frac{2x-1}{3x^2+x-2}\right)^2 \cdot \left\{\frac{2 \cdot (3x^2+x-2) - (2x-1)(6x+1)}{(3x^2+x-2)^2}\right\}$

$\frac{3(2x-1)^2 \cdot (6x^2+2x-4) - (12x^2+2x-6x-1)}{(3x^2+x-2)^4}$

$\frac{3(2x-1)^2 \cdot (6x^2+2x-4) - 12x^2+4x+1}{(3x^2+x-2)^4}$

$\frac{3(2x-1)^2 \cdot (-6x^2+6x-3)}{(3x^2+x-2)^4}$

$\frac{3(2x-1)^2 \cdot \{-3(2x^2-2x+1)\}}{(3x^2+x-2)^4}$

$\frac{-9(2x-1)^2(2x^2-2x+1)}{(3x^2+x-2)^4}$

29) $g(t) = \text{sen}^2(3t^2-1)$

$g'(t) = 2\,\text{sen}(3t^2-1) \cdot 6t$

$g'(t) = 2\,\text{sen}(3t^2-1) \cdot \cos(3t^2-1) \cdot 6t$

$g'(x) = 6t\,[2\,\text{sen}(3t^2-1) \cdot \cos(3t^2-1)]$

$g'(x) = 6t\,\text{sen}\,2(3t^2-1)$ — 2 sen a cos a = sen 2a

$g'(x) = 6t\,\text{sen}(6t^2-2)$

35) $f(x) = 4 \cdot \cos(\text{sen}\,3x)$

$f'(x) = -4\,\text{sen}(\text{sen}\,3x) \cdot (\cos 3x) \cdot 3$

$f'(x) = -12\cos 3x\ \text{sen}(\text{sen}\,3x)$

37) Ache uma equação da reta tangente à curva $y = (x^2-1)^2$ em cada um dos seguintes pontos $(-2,9), (-1,0), (0,1), (1,0)$ e $(2,9)$. Faça um esboço do gráfico e desenhe segmentos dessas retas tangentes nesses pontos desses

$y' = 2(x^2-1) \cdot 2x = 4x(x^2-1)$

$m(-2,9) = 4.(-2)((-2)^2-1)$
$= -8(4-1)$
$= -8\,3$
$= -24$

$m(2,9) = 4.2(2^2-1)$
$= 8\,3$
$= 24$

$m = (-1,0) = 4.(-1)((-1)^2-1)$
$= -4.0$
$= 0$

$m = (0,1) = 4.0(0^2-1)$
$= 0$

$m = (1,0) = 4.1(1^2-1)$
$= 4.0$
$= 0$

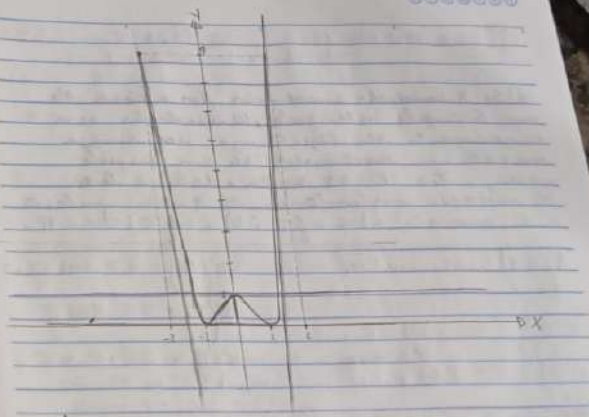
(-2, 9)
$y - y^0 = m(x - x^0)$
$y - 9 = -24(x+2)$
$y = -24x - 48 + 9$
$y = -24x - 39$
$24x + y + 39 = 0$

(2, 9)
$y - 9 = 24(x-2)$
$y = 24x - 48 + 9$
$y = 24x - 39$
$24x - y - 39 = 0$

(-1, 0)
$y - 0 = 0(x+1)$
$y = 0$

(0, 1)
$y - 1 = 0(x-0)$
$y = +1$

(1, 0)
$y - 0 = 0(x-1)$
$y = 0$



44) Uma onda produzida por um som simples tem a equação $P(t) = 0,003\,\text{sen}\,1800\pi t$, onde $P(t)$ dinas/cm² é a diferença entre a pressão atmosférica e a pressão do ar no tímpano, em t s. Ache a taxa de variação instantânea de $P(t)$ em relação a t em (a) $\frac{1}{9}$ s ; (b) $\frac{1}{8}$ s ; (c) $\frac{1}{7}$ s.

$P'(t) = 0,003\,\text{sen}\,1800\pi t$

$P'\left(\frac{1}{9}\right) = 0,003\,\text{sen}\,1800\pi \cdot \frac{1}{9}\,\pi$

$P(t) = 0,54\pi \cos 200\pi$

$P(t) = 0,54\pi \cdot 1 = 0,54\pi$

$P\left(\frac{1}{8}\right) = 0,54\pi$

$P\left(\frac{1}{7}\right) = 0,54\pi$

$f'\left(\frac{1}{7}\right) = 0,5\pi \cdot \cos 180\frac{\pi}{2}$ em $43,62\pi$

46) Se K unidades quadradas for a área de um Triângulo retângulo, 10 unidades será o comprimento da hipotenuza cos será a medida em radianos de um ângulo agudo, então $K = 25 \operatorname{sen} 2\alpha$, Ache a taxa de variação instantânea de K em relação a α, quando a) $\alpha = \frac{1}{6}\pi$; b) $\alpha = \frac{1}{4}\pi$; c) $\alpha = \pi$.

$K'\left(\frac{1}{6}\pi\right) = 25 \operatorname{sen} 2\alpha$

$K'\left(\frac{\pi}{6}\right) = 25 \cos 2\frac{\pi}{6} \cdot 2$

$K'\left(\frac{\pi}{6}\right) = 50 \cos \frac{\pi}{3}$

$K'\left(\frac{\pi}{6}\right) = 50 \cdot \frac{1}{2} = 25_{u}$

$K'\left(\frac{\pi}{4}\right) = 25 \cos 2\frac{\pi}{4} \cdot 2$

$K'\left(\frac{\pi}{4}\right) = 50 \cos \frac{\pi}{2}$

$K'\left(\frac{\pi}{4}\right) = 50 \cdot 0 = 0_{u}$

$K'(\pi) = 25 \cos 2\pi \cdot 2$

$K'(\pi) = 50 \cos 2\pi$

$K'(\pi) = 50 \cdot 1 = 50_{u}$

47) Com uma ida (3.6)

$y = \frac{1}{6}x^2 + 90 \qquad x = 7T + 85 \qquad T = 8$

$x' = 7$

$y' = 2 \cdot \frac{1}{6}x$

$y' = \frac{1}{3}x$

$\frac{Dy}{Dt} = \frac{1}{3} \cdot 7 = \frac{7}{3}x$

$\frac{Dy}{Dt} = \frac{7}{3} \cdot 141$

$\frac{Dy}{Dt} = 7 \cdot 47 = 329.$

Curva: $x = 7T + 85$

$x = 7 \cdot 8 + 85$

$x = 141$

A população de predadores variará a uma taxa de 329 indivíduos por ano.

3.7

de 1 a 28: fazer de 4 em 4

de 39 a 42: fazer de 2 em 2

fazer 43, 46, 48, 50.

$f(x) = n[g(x)]^{n-1} \cdot g'(x)$

01) $f(x) = 4x^{-1/2} + 5x^{-1/2}$

$f'(x) = 4 \cdot \frac{1}{2}x^{-\frac{3}{2}} + 5 \cdot -\frac{1}{2}x^{-\frac{3}{2}}$

$f'(x) = 2x^{-\frac{3}{2}} - \frac{5}{2}x^{-\frac{3}{2}}$

$f'(x) = x^{-\frac{3}{2}}\left(2 - \frac{5}{2}x^{-1}\right)$

5) $f(x) = (5-3x)^{2/3}$

$f'(x) = \frac{2}{3}(5-3x)^{-\frac{1}{3}} \cdot -3$

$f'(x) = -2(5-3x)^{-\frac{1}{3}}$

$f'(x) = \frac{-2}{(5-3x)^{\frac{1}{3}}}$

8) $f(x) = (5-2x^2)^{-\frac{1}{3}}$

$f'(x) = -\frac{1}{3}(5-2x^2)^{-\frac{4}{3}} \cdot (-4x)$

$f'(x) = \frac{4x}{3}(5-2x^2)^{-\frac{4}{3}}$

$f'(x) = \frac{4x}{3(5-2x^2)^{4/3}}$

13) $f(x) = (\operatorname{sen} 3x)^{-\frac{1}{2}}$

$f'(x) = -\frac{1}{2}(\operatorname{sen} 3x)^{-\frac{3}{2}} \cdot 3\cos 3x$

$f'(x) = -\frac{3}{2}\cos 3x\,(\operatorname{sen} 3x)^{-\frac{3}{2}}$

$f'(x) = \frac{-3\cos 3x}{2(\operatorname{sen} 3x)^{\frac{3}{2}}}$

17) $g(x) = \sqrt{\frac{2x-5}{3x+1}}$

$g'(x) = \left(\frac{2x-5}{3x+1}\right)^{\frac{1}{2}}$

$g'(x) = \frac{1}{2}\left(\frac{2x-5}{3x+1}\right)^{-\frac{1}{2}} \cdot \frac{(2x-5)'3 - 2(3x+1)}{(3x+1)^2}$

$g'(x) = \frac{1}{2}\left(\frac{2x-5}{3x+1}\right)^{-\frac{1}{2}} \cdot \frac{6x-15-6x-2}{(3x+1)^2}$

$g'(x) = \frac{1}{2}\left(\frac{2x-5}{3x+1}\right)^{-\frac{1}{2}} \cdot \frac{17}{(3x+1)^2}$

$g'(x) = \frac{1}{2}\left(\frac{3x+1}{2x-5}\right)^{\frac{1}{2}} \cdot \frac{17}{(3x+1)^2}$

$g'(x) = \frac{17}{2(2x-5)(3x+1)^3)^{\frac{1}{2}}}$

21) $g(t) = \sqrt{2t} + \sqrt{\frac{2}{t}}$

$g(t) = (2t)^{\frac{1}{2}} + \left(\frac{2}{t}\right)^{\frac{1}{2}}$

$g'(t) = \frac{1}{2}(2t)^{-\frac{1}{2}} \cdot 2 + \frac{1}{2}\left(\frac{2}{t}\right)^{-\frac{1}{2}}$

$g'(t) = 1(2t)^{-\frac{1}{2}} + \frac{1}{2}\left(\frac{t}{2}\right)^{\frac{1}{2}}$

$g'(t) = 1(2t)^{-\frac{1}{2}}$

25) $\frac{d}{dx}\left(\frac{\sqrt{x^2-1}}{x}\right)$

$\frac{d}{dx}\ \frac{x\cdot\frac{1}{2}(x^2-1)^{-\frac{1}{2}}\cdot 2x - (x^2-1)^{\frac{1}{2}}\cdot(1)}{x^2}$

$2^{-\frac{1}{2}}\cdot\frac{\left[x^2-(x^2-1)\right]}{x^2}\cdot\frac{1}{x^2\sqrt{x^2-1}}$

39) Ache uma equação da reta tangente à curva $y=\sqrt{x^2+9}$, no ponto $(4;5)$

$y=(x^2+9)^{\frac{1}{2}}$

$y'=\frac{1}{2}(x^2+9)^{-\frac{1}{2}}\cdot 2x$

$y'=2x\ (x^2+9)^{\frac{1}{2}}$

$y'=\frac{x}{\sqrt{x^2+9}}$ $\quad y=\frac{4}{\sqrt{4^2+9}}$ $\quad y=\frac{4}{\sqrt{16+9}}$ $\quad y=\frac{4}{\sqrt{25}}$ $\quad x=\frac{4}{5}$

$y-y^0=m(x-x^0)$

$y-5=\frac{4}{5}(x-4)$

$5y-25=4x-16$

$-4x+5y-9=0$

41) Ache uma equação da reta normal à curva $y=x\sqrt{16+x^2}$ na origem.

$y=x\sqrt{16+x^2}$

$y'=1\cdot x(x)\cdot(\sqrt{16+x^2})+x\cdot\frac{1}{2}(\sqrt{16+x^2})$

$y'=\frac{1}{2}\cdot\sqrt{16+x^2}+x\cdot\frac{1}{2}(16+x^2)^{-\frac{1}{2}}\cdot 2x$

$y'=\sqrt{16+x^2}+x\cdot\frac{1}{2\sqrt{16+x^2}}\cdot 2x$

$y'=\sqrt{16+x^2}+x\cdot\frac{2x}{2\sqrt{16+x^2}}$

$y'=\sqrt{16+x^2}+\frac{x^2}{\sqrt{16+x^2}}$

coeficiente A ($x=0$)

$m_p=\sqrt{16+0^2}+\frac{0^2}{\sqrt{16+0^2}}$

$m_p=\sqrt{16}+\frac{0}{\sqrt{16}}$

$m=4+\frac{0}{4}$

$m=4+0$

$m=4_{11}$

$m_{reta}\cdot m_p=-1$

$m_{reta}\cdot 4=-1$

$m_{reta}=-\frac{1}{4}$

$y-y^0=m(x-x^0)$

$y-0=-\frac{1}{4}(x-0)$

$4y=-x$

$x+4y=0$

43) Um objeto move-se ao longo de uma reta, de acordo com a equação de movimento $s = \sqrt{4t^2+3}$, com $t \geq 0$. Ache o valor de t para o qual a magnitude da velocidade instantânea é: (a) 0 ; (b) 1 ; (c) 2.

$s = \sqrt{4t^2+3}$

$s'(t) = (4t^2+3)^{\frac{1}{2}}$

$s'(t) = \frac{1}{2}(4t^2+3)^{-\frac{1}{2}} \cdot 8t$

$s'(t) = \frac{4t}{\sqrt{4t^2+3}}$

Para velocidade = tomou

P/(a)

$\frac{4t}{\sqrt{4t^2+3}} = 0$

$4t = 0 \cdot \sqrt{4t^2+3}$

$4t = 0$

$t = \frac{0}{4}$

$t = 0$

P/(1)

$\frac{4t}{\sqrt{4t^2+3}} = 1$

$(4t)^2 = \left(\sqrt{4t^2+3}\right)^2$

$16t^2 = 4t^2 + 3$

$16t^2 - 4t^2 = 3$

$12t^2 = 3$

$t^2 = \frac{3}{12}$

$t^2 = \frac{1}{4}$

$t = \sqrt{\frac{1}{4}}$

$t = \frac{1}{2}$

P/(2)

$\frac{4t}{\sqrt{4t^2+3}} = 2$

$4t = 2\sqrt{4t^2+3}$

$\frac{4t}{2} = \sqrt{4t^2+3}$

$(2t)^2 = \left(\sqrt{4t^2+3}\right)^2$

$4t^2 = 4t^2 + 3$

$4t^2 - 4t^2 = 3$

$0 = 3$

isso é falso nenhum valor de t

46) A quantia em dinheiro em certo total de produção de x unidades de certa mercadoria é dada por $C(x) = 40 + 3x + 9\sqrt{2x}$. Ache (a) o custo marginal quando 50 unidades são produzidas e (b) o número de unidades produzidas quando o custo marginal é $ 4,50.

$C(x) = 40 + 3x + 9\sqrt{2x}$

$C'(x) = 40 + 3x + \frac{9}{\sqrt{2x}}$

$C'(x) = 3 + \frac{9}{\sqrt{2x}}$

$D_x = 9\sqrt{2x}$

$D_x = (9 \cdot 2x)^{\frac{1}{2}}$

$D_x = \frac{1}{2}(9 \cdot 2x)^{-\frac{1}{2}} \cdot 2$

$D_x = \frac{9}{\sqrt{2x}}$

(A) P/ $x = 50$

$C(x) = 3 + \frac{9}{\sqrt{2 \cdot 50}}$

$C(x) = 3 + \frac{9}{10}$

$C(x) = \frac{30+9}{10}$

$C(x) = \frac{39}{10}$

$C(x) = 3,9$

$C = 3 + \frac{9}{\sqrt{2 \cdot 7}}$

$C = 3 + \frac{9}{6}$

$CX = \frac{18+9}{6}$ $C = \frac{27}{6}$ $C = 4,5$

(B) P/ $x = 4,50$

$C(x) = 3 + \frac{9}{\sqrt{2x}}$

$4,50 = 3 + \frac{9}{\sqrt{2x}}$

$4,5 - 3 = \frac{9}{\sqrt{2x}}$

$1,5 = \frac{9}{\sqrt{2x}}$

$\sqrt{2x} \cdot 1,5 = 9$

$\sqrt{2x} = \frac{9}{1,5}$

$\left(\sqrt{2x}\right)^2 = (6)^2$

$2x = 36$

$x = \frac{36}{2}$

$x = 18$

48) A produção diária de uma dada fábrica é de $P(x)$ unidades, sendo o capital investido por dia x milhares de uma unidade monetária $ e $P(x) = 200\sqrt{2x+1}$, se a capitalização corrente for de $ 760.000, use a derivada para estimar a variação na produção diária, se o capital investido for aumentado em $ 1000.

$P(x) = 200 \cdot \sqrt{2x+1}$

$P'(x) = 200 (2x+1)^{\frac{1}{2}}$

$P'(x) = \frac{1}{2} \cdot 200(2x+1)^{-\frac{1}{2}} \cdot 2$

$P'(x) = \frac{200}{\sqrt{2x+1}}$

P/ 760.000

$P'(760.000) = \frac{200}{\sqrt{2.760.000+1}}$

$P'(760.000) = \frac{200}{\sqrt{1.522.000+1}}$

$P'(760.000) = \frac{200}{390}$

$\boxed{P'(760000) = 0,511}$

P/ 761.000

$P'(761.000) = \frac{200}{\sqrt{2.761.000+1}}$

$P'(761.000) = \frac{200}{\sqrt{1522.000+1}}$

$P'(761.000) = \frac{200}{40,01}$

$\boxed{P'(761.000) = 4,99}$

50) Dada $f(u) = \frac{1}{u^2}$ e $g(x) = \frac{\sqrt{x}}{\sqrt{2x^2-6x+1}}$, ache a derivada de $f \circ g$ de duas maneiras: a) usando primeiro $(f \circ g)(x)$ e depois $(f \circ g)'(x)$. b) usando a regra da cadeia.

$f(u) = \frac{1}{u^2}$

$f \circ g(x) = \frac{1}{\left(\frac{\sqrt{x}}{\sqrt{2x^2-6x+1}}\right)^2}$

$f \circ g(x) = \frac{1}{\frac{x}{2x^2-6x+1}}$

$f \circ g(x) = \frac{1}{x} \cdot \frac{2x^2-6x+1}{x}$

$\boxed{f \circ g(x) = 2x^2 - 6 + x^{-1}}$

$f \circ g(x) = 2x^2 - 6 + x^{-1}$

$f \circ g'(x) = 4x - x^{-2}$

$\boxed{f \circ g(x) = 4x - \frac{1}{x^2}}$

3.8 Derivação Implícita

de 01 a 22: fazer 1a coluna 4
de 33 a 36: fazer 01 a coluna 2

1) $x^2 + y^2 = 16$

A) $x^2 + y^2 = 16$

$y = \pm\sqrt{16 - x^2}$

B) $f_1 y = \sqrt{16-x^2}$ ou C) $f_2 y = -\sqrt{16-x^2}$

D) $f_1'(x) = \sqrt{16-x^2}'$

$f_1'(x) = (16-x^2)^{\frac{1}{2}}$

$f_1'(x) = \frac{1}{2}(16-x^2)^{-\frac{1}{2}} \cdot 2x$

$f'(x) = x(16-x^2)^{-\frac{1}{2}}$

$f'(x) = \frac{x}{\sqrt{16-x^2}}$

E) $\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}9$

$2x + 2y \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{2x}{2y}$

$\frac{dy}{dx} = -\frac{x}{y}$

$f_2'(x) = -(16-x^2)^{\frac{1}{2}}$

$f_2'(x) = -\frac{1}{2}(16-x^2)^{-\frac{1}{2}}(-2x)$

$f_2'(x) = x(16-x^2)^{-\frac{1}{2}}$

$f_2'(x) = \frac{x}{\sqrt{16-x^2}}$

Logo $\frac{dy}{dx} = -\frac{x}{\sqrt{16-x^2}} \Rightarrow -\frac{x}{y} \Rightarrow \frac{x}{y}$ $f_1' = f_2''$

$\frac{dy}{dx} = -\frac{x}{\sqrt{16-x^2}} \Rightarrow \frac{x}{-y} \Rightarrow -\frac{x}{y}$

então $\frac{dy}{dx} = -\frac{x}{y}$

5) $\frac{1}{x} + \frac{1}{x} = 1$ $\quad \frac{d}{dx}(x)^{-1} + \frac{1}{x}(x)^{-1} = \frac{d}{dx}1$

$-x^{-2} - y^{-2} \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x^{-2}}{-y^{-2}}$

$\frac{dy}{dx} = \frac{1}{x^2} \cdot \frac{y^2}{1}$

$\frac{dy}{dx} = \frac{y^2}{x^2}$

9) $x^2 \cdot y^2 = x^2 + y^2$ $\quad \frac{d}{dx}(x^2)\cdot y^2 + x^2 \cdot \frac{d}{dx}(y^2) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$

$2x \cdot y^2 + x^2 \cdot 2y \cdot \frac{dy}{dx} = 2x + 2y \cdot \frac{dy}{dx}$

$2xy^2 + 2x^2y \cdot \frac{dy}{dx} = 2x + 2y \cdot \frac{dy}{dx}$

$xy^2 + x^2y \cdot \frac{dy}{dx} = x + y\frac{dy}{dx}$

$x^2y\frac{dy}{dx} - y\frac{dy}{dx} = x - xy^2$

$\frac{d}{dx}(x^2y - y) = x - xy^2$

$\boxed{\frac{dy}{dx} = \frac{x - xy^2}{x^2y - y}}$

13) $\sqrt[3]{x} + \sqrt[3]{xy} = 4y^2$

$\frac{d}{dx}(\sqrt[3]{x}) + \frac{d}{dx}(\sqrt[3]{xy}) = \frac{d}{dx} 4y^2$

$\frac{d}{dx}(x^{\frac{1}{3}}) + \frac{d}{dx}(xy)^{\frac{1}{3}} = 8y \frac{dy}{dx}$

$\frac{1}{3} x^{-\frac{2}{3}} + \frac{1}{3}(xy)^{-\frac{2}{3}} \cdot (x + y\frac{dx}{dx}) = 8y\frac{dy}{dx}$

15) $\sqrt{xy} + 2x = \sqrt{y}$ $\quad \frac{d}{dx}\sqrt{xy} + \frac{d}{dx} 2x = \frac{d}{dx}\sqrt{y}$

$\frac{d}{dx}\sqrt{xy}$

$\frac{1}{2}(xy)^{-\frac{1}{2}} \cdot (y + x\frac{dy}{dx})$

$\frac{1}{2\sqrt{xy}}\left(y + x\frac{dy}{dx}\right)$

$\frac{d}{dx} \cdot \frac{1}{2}(y)^{-\frac{1}{2}}$

$\frac{d}{dx} \frac{1}{2\sqrt{y}}$

$\frac{1}{2\sqrt{xy}}\left(y + x\frac{dy}{dx}\right) = \frac{1}{2\sqrt{y}}\frac{dy}{dx}$

$\frac{y + xD}{2\sqrt{xy}} + 2 = \frac{1}{2\sqrt{y}} \cdot D$

$y + xD + 2 = \frac{1}{2\sqrt{y}} \cdot 2\sqrt{xy}$

$xD = \frac{1}{2\sqrt{y}} \cdot 2\sqrt{xy} - y - 2$

$xD = \frac{2\sqrt{xy}}{2\sqrt{y}} - y - 2$

$xD = \frac{\sqrt{xy} - \sqrt{y^2} - 2\sqrt{y}}{2\sqrt{y}}$

$D = \frac{\sqrt{x} - \sqrt{y} \cdot 2\sqrt{y} \cdot x}{2\sqrt{y}}$

$= \frac{x\sqrt{x} - x\sqrt{y} - 2x\sqrt{y}}{2\sqrt{y}}$

19) $x^2y^3 = x^4 - y^4$

$2x \cdot y^3 + 3y^2 \frac{dy}{dx} \cdot x^2 = 4x^3 - 4y^3 \frac{dy}{dx}$

$2xy^3 + 3y^2 \frac{dy}{dx} x^2 = 4x^3 - 4y^3 \frac{dy}{dx}$

$3y^2 \frac{dy}{dx} x^2 - 4x \cdot 4y^3 \frac{dy}{dx} = 2xy^3$

$3y^2 \frac{dy}{dx} x^2 + 4y^3 \frac{dy}{dx} = 4x^3 - 2xy^3$

$y^2 \frac{dy}{dx}(3x^2 + 4y) = 4x^3 - 2xy^3$

$\frac{dy}{dx} = \frac{4x^3 - 2xy^3}{y^2(3x^2 + 4y)}$

21) $\sec^2 x + \operatorname{cosec}^2 y = 4$

$2\sec x \tan x - \operatorname{cosec} y \cot y \frac{dy}{dx} = 0$

$-\operatorname{cosec}^2 y \cot y \frac{dy}{dx} = -2\sec x \tan x$

$\frac{dy}{dx} = \frac{2\sec^2 x \tan x}{\operatorname{cosec}^2 y \cot y}$

$\frac{dy}{dx} = \frac{\sec x \tan x}{\operatorname{cosec}^2 y \cot y}$

27. $(x+y)^2 - (x-y)^2 = x^3 + y^3$

$\frac{dy}{dx}\left[(x+y)^2 - (x-y)^2\right] = \frac{dy}{dx}(x^3+y^3)$

$\frac{dy}{dx}(x+y)^2 - \frac{dy}{dx}(x-y)^2 = 3x^2 + 3y^2\frac{dy}{dx}$

$2(x+y)\left(1+\frac{dy}{dx}\right) - 2(x-y)\left(1-\frac{dy}{dx}\right) = 3x^2 + 3y^2\frac{dy}{dx}$

$2 + 2\frac{dy}{dx} + 2 - 2\frac{dy}{dx} = 3x^2 + 3y^2\frac{dy}{dx}$

$4 = 3x^2 + 3y^2\frac{dy}{dx}$

$\frac{dy}{dx}(9x - 3y^2) = 3x^2 - 4y$

$\frac{dy}{dx} = \frac{3x^2 - 4y}{4x - 3y^2}$

3300 36 Lacoda 2

33) Ache uma equação da reta tangente à curva $16x^4 + y^4 = 32$ no ponto $(1, 2)$

$16x^4 + y^4 = 32$

$64x^3 + 4y^3\frac{dy}{dx} = 0$

$\frac{dy}{dx}4y^3 = -64x^3$

$\frac{dy}{dx} = -\frac{64x^3}{4y^3} = -\frac{16x^3}{y^3}$

$m = -\frac{16 \cdot 1^3}{2^3}$

$m = -\frac{16}{8} = -2$

$y - y^0 = m(x - x^0)$

$y - 2 = -2(x - 1)$

$y - 2 = -2x + 2$

$\boxed{2x + y - 4 = 0}$

35) Ache uma equação da reta normal à curva $x^2 + xy + y^2 - 3y = 10$ no ponto $(2, 4)$

$x^2 + xy + y^2 - 3y = 10$

$2x + 1\cdot x\frac{dy}{dx} + 2y\frac{dy}{dx} - 3\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x + 2y - 3) = -2x - y$

$\frac{dy}{dx} = \frac{-2x - y}{x + 2y - 3}$

$m = \frac{-2 \cdot 2 - 4}{2 + 2 \cdot 4 - 3}$

$m = -\frac{8}{5}$

$y - 4 = \frac{5}{8}(x - 2)$

$m_1 \cdot m_2 = -1$

$m_2 = -\frac{8}{5} = -1$

$m_2 = -1 \cdot \frac{5}{-8}$

$m_2 = \frac{5}{8}$

$8y - 32 = 5x - 10$

$-5x + 8y - 22 = 0$

3.10

De 1 ao 16 ficam o conde 4
De 20 ao 24 ficam 2
De 25 ao 28 ficam 1 e conde 2
De 35 ao 39 ficam 1

① $f(x) = x^3 - 2x^2 + x$

$f'(x) = 3x^2 - 4x + 1$

$f''(x) = 6x - 4$

4) $g(t) = t^3 - t^2 + t$

$g'(t) = 3t^2 - 2t + 1$

$g''(t) = 6t - 2$

9) $\rho(t) = 4\cos t^2$

$4\rho(t) = -4\operatorname{sen} t^2 \cdot 2t$

$\rho(t) = 4 \cdot (2t - \operatorname{sen} t^2)$

$\rho'(t) = -8t + \operatorname{sen} t^2$

$\rho''(t) = -8 \cdot t \cdot (+\operatorname{sen}(t^2))$

$\rho'(t) = -8\left(\frac{d}{dt}\operatorname{sen}(t^2)\right) + \frac{d}{dt}(\operatorname{sen}(t^2) \cdot t$

$\rho'(t) = -8(\operatorname{sen} t^2 + 2t^2 \cos t^2)$

$\rho''(t) = -16t^2 \cos t^2 - 8\operatorname{sen} t^2$

13) $g(x) = \frac{x^2}{x^2+4}$

$g'(x) = \frac{2x \cdot (x^2+4) - x^2 \cdot 2x}{(x^2+4)^2}$

$g'(x) = \frac{2x \cdot (x^2+4-x^2)}{(x^2+4)^2}$

$g'(x) = \frac{2x^3 + 8x - 2x^3}{(x^2+4)^2}$

$g'(x) = \frac{8x}{(x^2+4)^2}$

$g''(x) = \frac{8x}{(x^2+4)^2}$

$8g''(x) = \frac{x}{(x^2+4)^2}$

$8g'(x) = \frac{1 \cdot (x^2+4)^2 + x \cdot 2(x^2+4) \cdot 2x}{((x^2+4)^2)^2}$

$8g'(x) = \frac{(x^2+4) - 4x \cdot (x^2+4)x^2}{(x^2+4)^4}$

$8g''(x) = \frac{(x^2+4)(x^2+4-4x^2)}{(x^2+4)^4}$

$8g''(x) = \frac{(x^2+4)(-3x^2+4)}{(x^2+4)^4}$

$g''(x) = 8 \cdot \frac{(-3x^2+4)}{(x^2+4)^3}$

21) Achar $D^3(2 + \operatorname{tg} 3x)$

$D(x) = (2 + \operatorname{tg} 3x)$

$D'(x) = 2 \sec^2(3x) \cdot 3$

$D''(x) = 6 \sec^2(3x)$

$D''(x) = 12 \sec 3x \cdot (\sec 3x \tan(3x) \cdot 3)$

$D''(x) = 36 \sec^2(3x) \tan(3x)$

$D'''(x) = 36 \cdot 2 \sec(3x) \cdot \tan(3x) \cdot 3 + 3 \sec^2(3x)$

$D'''(x) = 216 \sec^2(3x) \tan^2(3x) + 108 \sec^4(3x)$

$D'''(x) = 108 \sec^2(3x)(3\operatorname{tg}^2 3x + 1)$

23) Achar $\rho^{(5)}(x)$ se $\rho(x) = \cos 2x - \operatorname{sen} 2x$

$\rho = \cos(2x) - \operatorname{sen}(2x)$

$\rho' = -2\operatorname{sen}(2x) - 2\cos(2x)$

$\rho'' = -2[2\cos(2x)] - 2[-2\operatorname{sen}(2x)]$

$\rho'' = -4\cos(2x) + 4\operatorname{sen}(2x)$

$\rho''' = -4[-2\operatorname{sen}(2x)] + 4[2\cos(2x)]$

$\rho''' = 8\operatorname{sen}(2x) + 8\cos(2x)$

$\rho^4 = 8[2\cos(2x)] + 8[-2\operatorname{sen}(2x)]$

$\rho^4 = 16\cos(2x) - 16\operatorname{sen}(2x)$

$\rho^5 = d_x(16\cos(2x) - 16\operatorname{sen}(2x)$

$\rho^5 = 16[-2\operatorname{sen}(2x)] - 16[2\cos(2x)]$

$\rho^5 = -32\operatorname{sen}(2x) - 32\cos(2x)$

$\rho^5 = -32[\operatorname{sen}(2x) + \cos(2x)]$

25) Dado $x^2 + y^2 = 1$, mostre que $\frac{d^2y}{dx^2} = \frac{1}{y^3}$

$x^2 + y^2 = 1$

$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(1)$

$2x + 2y \cdot \frac{dy}{dx} = 0$

$2y \frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y}$

$\frac{dy}{dx} = -\frac{x}{y}$

$\frac{d^2y}{dx^2} = -\frac{x' \cdot y - x \cdot 1 \cdot \frac{dy}{dx}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{y - x\frac{dy}{dx}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{y - x \cdot \left(-\frac{x}{y}\right)}{y^2}$

$\frac{d^2y}{dx^2} = \frac{y + \frac{x^2}{y}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{\frac{y^2 + x^2}{y}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{y^2 + x^2}{y} \cdot \frac{1}{y^2}$

$\frac{d^2y}{dx^2} = \frac{y^2 + x^2}{y^3}$ → = 1

$\boxed{\frac{d^2y}{dx^2} = \frac{1}{y^3}}$

27) Dado $x^3 + y^3 = 1$, mostre que $\frac{d^2y}{dx^2} = -\frac{2x}{y^5}$

$x^3 + y^3 = 1$

$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx} 1$

$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 0$

$3y^2 \frac{dy}{dx} = -3x^2$

$\frac{dy}{dx} = -\frac{3x^2}{3y^2}$

$\frac{dy}{dx} = -\frac{x^2}{y^2}$

$\frac{d^2y}{dx^2} = -\frac{x' \cdot y^2 - x^2 \cdot 2y \frac{dy}{dx}}{y^4}$

$\frac{d^2y}{dx^2} = \frac{2xy^2 - 2x^2y \cdot \frac{dy}{dx}}{y^4}$

$\frac{d^2y}{dx^2} = \frac{2xy^2 - 2x^2y \cdot \left(-\frac{x^2}{y^2}\right)}{y^4}$

$\frac{d^2y}{dx^2} = \frac{2xy^2 + \frac{2x^4}{y}}{y^4}$

$\frac{d^2y}{dx^2} = \frac{\frac{2xy^3 + 2x^4}{y}}{y^4}$

$\frac{d^2y}{dx^2} = \frac{2xy^3 + 2x^4}{y} \cdot \frac{1}{y^4}$

$\frac{d^2y}{dx^2} = \frac{2xy^3 + 2x^4}{y^5}$

$\frac{d^2y}{dx^2} = \frac{2x(x^3 + y^3)}{y^5}$ → = 1

$\boxed{\frac{d^2y}{dx^2} = \frac{2x}{y^5}}$

36) $s = \frac{1}{3}t^3 - 6t^2 + 3t - 4, \quad t \geq 0$

$s = 6t^2 - 12t + 3$

$a(t) = 6t - 12t + 3$

$a(t) = 12t - 12$

Velocidade = $|v(t)| = |6t^2 - 12t| =$

aceleração = $a(t) = 12t - 12$

Seção 7.3

do 1 ao 26: fazer o 1 e o caso os

① $f(x) = \ln(4+5x)$

$\frac{d}{dx}[\ln(4+5x)]$

$\frac{d}{dx} = \frac{1}{4+5x} \cdot 5$

$\frac{d}{dx} = \frac{5}{4+5x}$

$\ln(u)$

$(u) = 4+5x$

$\ln(x) = \frac{1}{u} \cdot \frac{du}{dx}$

$\frac{d}{dx}(u) = 4+5x$

$\frac{du}{dx} = 5$

dy

4) $f(x) = \ln(8-2x)$

$\frac{d}{dx}[\ln(8-2x)]$

$\frac{d}{dx} = \frac{1}{8-2x} \cdot -2$

$\frac{d}{dx} = -\frac{2}{8-2x}$

$\frac{d}{dx} = -\frac{1}{4-x}$

$(u) = 8-2x$

$\ln(u) = \frac{1}{u} \cdot \frac{du}{dx}$

$\frac{d}{dx}(u) = 8-2x$

$\frac{d}{du}(u) = -2$

11) $f(y) = \ln(\operatorname{sen} 5y)$

$f'(y) = \frac{1}{\operatorname{sen} 5y} \cdot 5\cos 5y$

$f'(y) = \frac{5\cos 5y}{\operatorname{sen} 5y}$

$\ln(u) = \frac{1}{u} \cdot \frac{du}{dx}$

$(u) = \operatorname{sen} 5y$

$(u) = 5\cos 5y$

15) $g(x) = \ln(\sec 2x + \operatorname{tg} 2x)$

$g'(x) = \frac{1}{\sec 2x + \operatorname{tg} 2x} \cdot [2\sec 2x(\operatorname{tg} 2x + \sec 2x)]$

$g'(x) = \frac{2\sec 2x(\operatorname{tg} 2x + \sec 2x)}{\sec 2x + \operatorname{tg} 2x}$

$g'(x) = 2\sec 2x$

$\ln(u) = \frac{1}{u} \cdot \frac{du}{dx}$

$(u) = \sec 2x + \operatorname{tg} 2x$

$(u) = 2\sec 2x \operatorname{tg} 2x + \sec^2 2x$

$u' = [2\sec 2x(\operatorname{tg} 2x + \sec 2x)]$

(17) $f(x) = \ln\sqrt{\operatorname{tg} x}$

$f(x) = \ln(\operatorname{tg} x)^{\frac{1}{2}}$

$f'(x) = \frac{1}{\sqrt{\operatorname{tg} x}} \cdot \frac{\sec^2 x}{2\sqrt{\operatorname{tg} x}}$

$f'(x) = \frac{\sec^2 x}{2(\sqrt{\operatorname{tg} x} \cdot \sqrt{\operatorname{tg} x})}$

$f'(x) = \frac{\sec^2 x}{2\operatorname{tg} x}$

$\ln u = \frac{1}{u} \cdot \frac{du}{dx}$

$(u) = (\operatorname{tg} x)^{\frac{1}{2}}$

$u = \frac{1}{2}(\operatorname{tg} x)^{\frac{1}{2}} \cdot \sec^2$

$u = \frac{1}{2} \cdot \frac{1}{\sqrt{\operatorname{tg} x}} \cdot \sec^2$

$u = \frac{1}{2\sqrt{\operatorname{tg} x}} \sec^2 x$

(21) $h(x) = \frac{x}{\ln x}$

$g(x) = \ln$

$\frac{\varphi'(x)}{g'(x)} = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$

$h'(x) = \frac{1 \cdot \ln(x) - 1}{(\ln x)^2}$

$\frac{f'(x)}{g'(x)} = \frac{1 \ln x - x \cdot \frac{1}{x}}{(\ln x)^2}$

$h'(x) = \frac{\ln(x) - 1}{(\ln(x))^2}$

$\frac{\varphi'(x)}{g'(x)} = \frac{1 \ln(x) - \frac{1}{\ln(x)^2}}{\ln(x)^2}$

Seção 7.5

fazer do 01 ao 16 - fazer 1 a cada 4

1) $y = e^{5x}$

$\frac{d}{dx} = e^{5x} \cdot \frac{d}{dx}(5x)$

$\frac{d}{dx} = e^{5x} \cdot 5$

(5) $y = e^{\cos x}$

$\frac{d}{dx} = e^{\cos x} \cdot \frac{d}{dx} \cos x$

$\frac{d}{dx} = e^{\cos x} \cdot -\operatorname{sen} x$

$\frac{d}{dx} = -e^{\cos x} \cdot \operatorname{sen} x$

(10) $y = e^{e^x}$

$\frac{d}{dx} = e^{e^x} \cdot \frac{d}{dx} e^x$

$a^b \cdot a^c = a^{b+c}$

$\frac{d}{dx} = e^{e^x} \cdot e^x$

$\frac{d}{dx} = e^{e^x + x}$

(14) $y = \ln(e^x + e^{-x})$

$\frac{d}{dx} = \frac{1}{e^x + e^{-x}} \cdot e^x \cdot 1 + e^{-x} \cdot -1$

$\frac{d}{dx} = \frac{1}{e^x + e^{-x}} \cdot e^x - e^{-x}$

$\frac{d}{dx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Seção 7.6

fazer 02, 03, 4, 9, 10

(2) $\varphi(x) = 6^{-3x}$

$\varphi'(x) = 6^{-3x}$

$\varphi'(x) = 216^{-x}$

$f(x) = \left(\frac{1}{216}\right)^x$

③ $\varphi(T) = 4^{3T^2}$

$\varphi'(T) = 4^{3T^2} \cdot \ln(4) \cdot 6T$

$\varphi'(x) = 6T \ln(4) 4^{3T^2}$

$a = 4$
$u(t) = 3T^2$
$u'(t) = 6T$

④ $g(x) = 10^{(x^2 - 2x)}$

$g'(x) = 10^{x^2-2x} \cdot \ln 10 \cdot 2x - 2$

$g'(x) = (2x-2) \ln(10)\, 10^{x^2-2x}$

$a = 10$
$u(t) = x^2 - 2x$
$u'(t) = 2x - 2$

$a^{u(x)} \cdot \ln(a) \cdot u'(x)$

⑨ $\ln(x) = \dfrac{\log_{10} x}{x}$ $\quad f(x)$ / $g(x)$

$f(T) = \dfrac{1}{x \ln 10}$

$g'(x) = 1$

$\ln'(x) = \dfrac{\frac{1}{x \ln 10} \cdot x - \log_{10} x \cdot 1}{x^2}$

$\ln'(x) = \dfrac{\frac{1}{\ln 10} - \log_{10} x}{x^2}$

$\log_{10} x = \dfrac{\ln x}{\ln 10}$

$\ln'(x) = \dfrac{\frac{1}{\ln 10} - \frac{\ln x}{\ln 10}}{x^2}$

$\ln'(x) = \dfrac{1 - \ln x}{x^2 \ln 10}$

$\ln'(x) = \dfrac{1 - \ln x}{\ln 10} \cdot \dfrac{1}{x^2}$

⑩ $\varphi(T) = \log_{10}\left(\dfrac{T}{T+1}\right)$ $\quad a / b$

$\dfrac{1}{u \ln(a)} \cdot \dfrac{du}{dx}$

$\log_a(b) = \dfrac{\ln(b)}{\ln(a)}$

$\log_a(b) = \left(\dfrac{\ln \frac{T}{T+1}}{\ln 10}\right)$

$\varphi'(T) = \dfrac{1}{T \ln(10)} - \dfrac{1}{(T+1) \ln(10)}$

$\varphi'(T) = \dfrac{1}{\ln(10)} \left(\dfrac{1}{T} - \dfrac{1}{T+1}\right)$

$\varphi'(T) = \dfrac{1}{\ln(10)} \left(\dfrac{(T+1) - T}{T(T+1)}\right)$

$\varphi'(T) = \dfrac{1}{\ln(10)} \left(\dfrac{1}{T(T+1)}\right)$

$\varphi'(T) = \dfrac{1}{T(T+1) \ln(10)}$